# Strongly coupled gauge theories: In and out of the conformal window

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BNL Lunch Seminar
Oct 24, 2013

In collaboration with A. Cheng, G. Petropoulos and D. Schaich
ArXiv:1301.1355,1310.1124

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- could the SMS be strongly coupled composite?
  - yes, until proven otherwise
- why is it so hard for lattice simulations?
  - this is not QCD!
- 2 methods & some results
  - spectral density of the Dirac operator eigenmodes
  - finite size scaling with corrections

# July 4th

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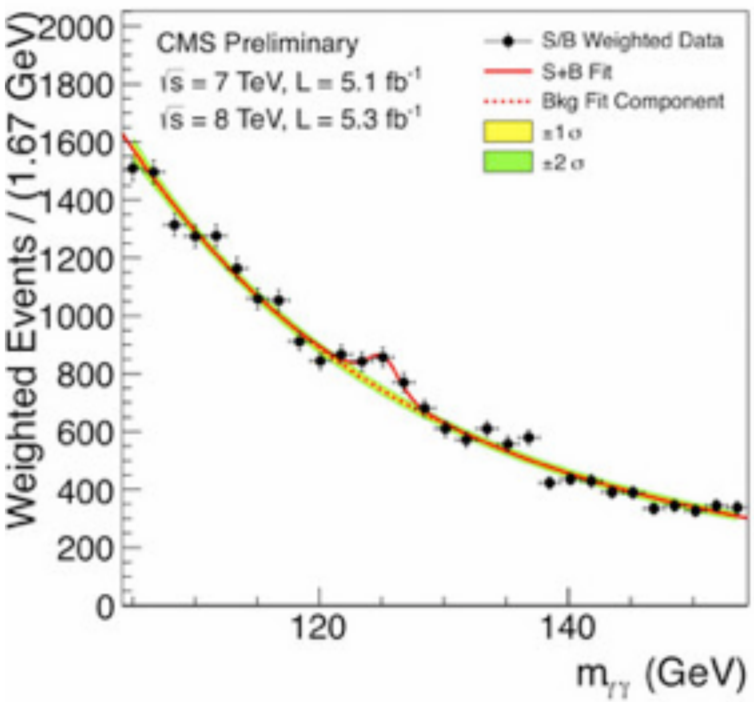



0++ scalar at 126 GeV :

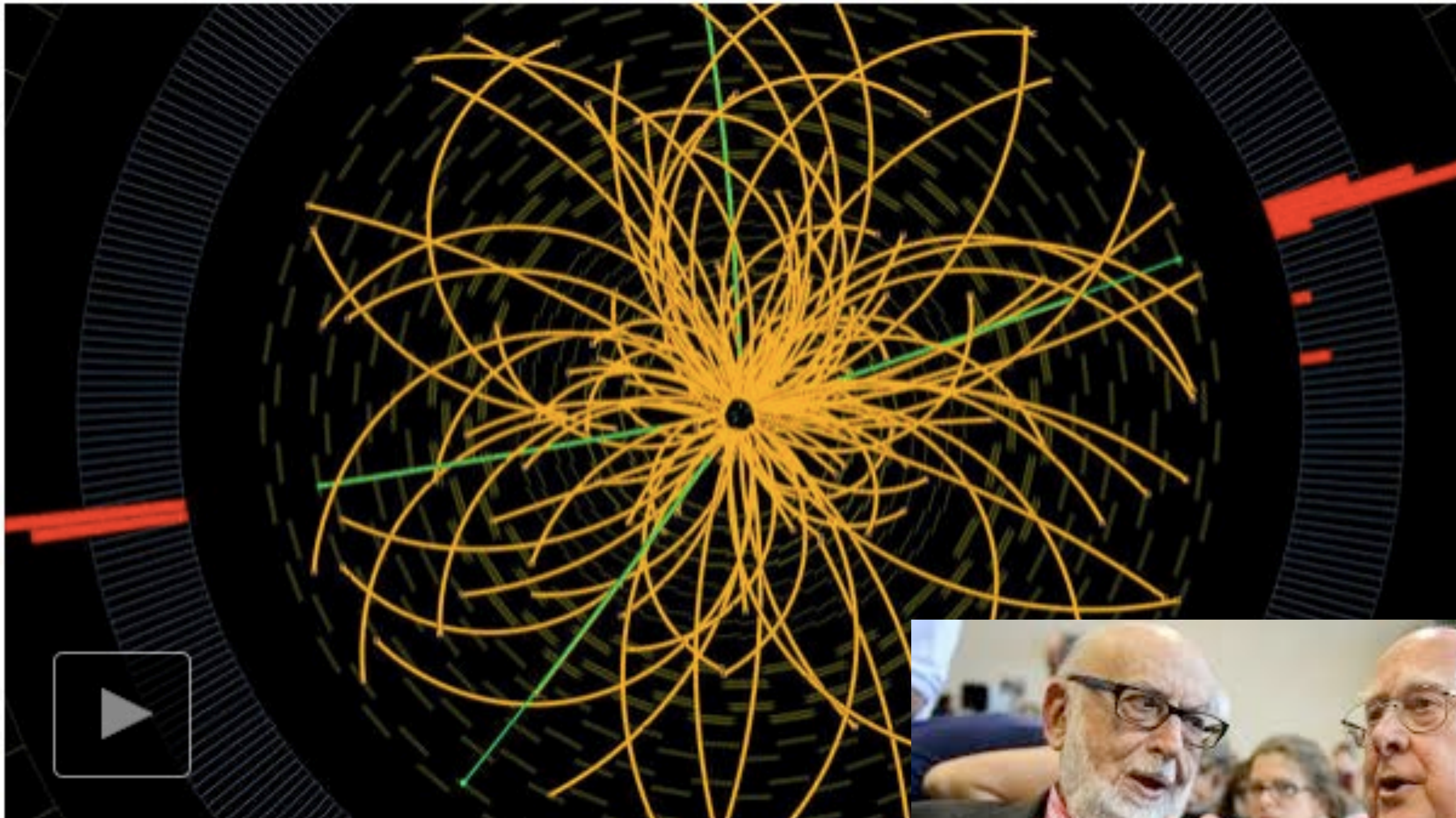
Standard Model like

- no sign of new TeV-scale physics!

Oct 8 2013

# For Nobel, They Can Thank the ‘God Particle’

## Higgs and Englert Are Awarded Nobel Prize in Physics



**Higgs Boson Particle Theory Wins Nobel:** Dennis Overbye, a Times s
particle's significance — and insignificance — in understanding our universe.

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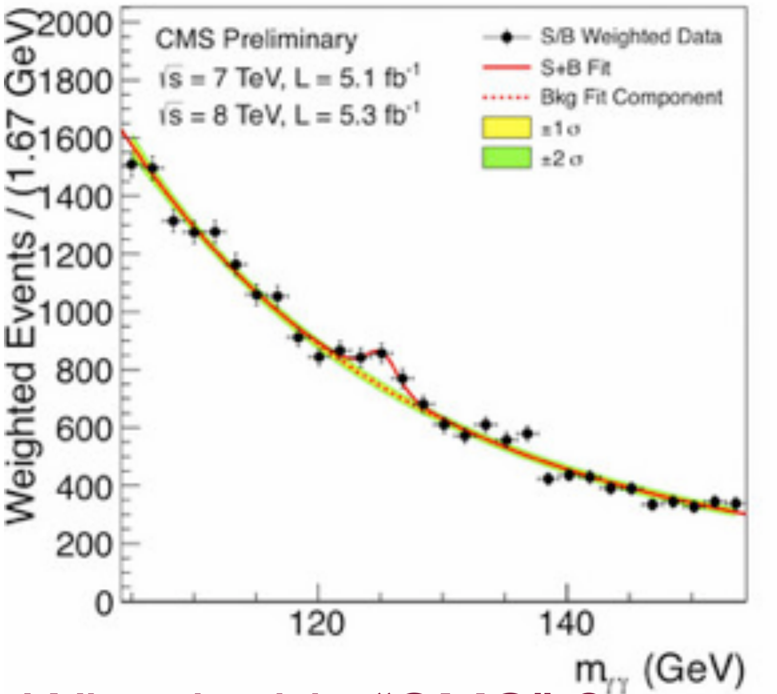


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What is this "SMS" ?

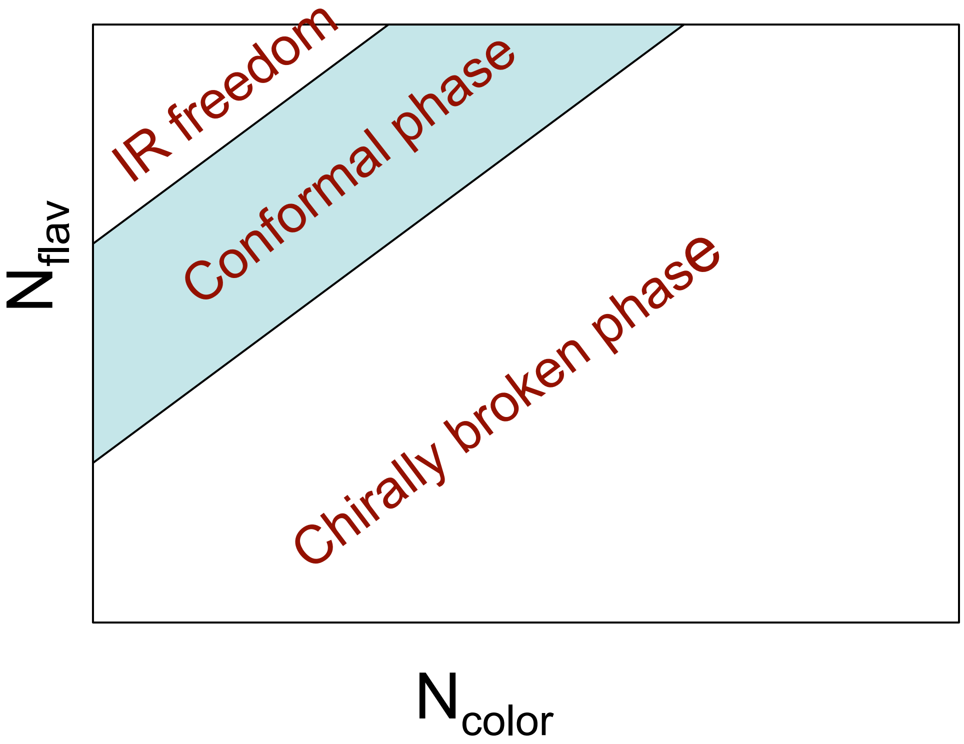
- Elementary scalar? and no new physics :
  give up naturalness and deal with fine tuning
- SUSY? SMS is uncomfortably heavy
- Composite? SMS is uncomfortably light
  find strongly interacting model with light scalar

# Composite Higgs:
## Strongly coupled meson-like fermion state

- Scaled-up QCD (technicolor) models are out (were ruled out decades ago)!
  – EW measurements are violated (mainly because $g^2$ runs too fast)
- Walking TC models: If exist, can solve most these problems;
  - Do they have a light Standard Model like scalar? Could be:
    - dilaton of spontaneously broken conformal symmetry
    - pseudo-Goldstone of expanded flavor symmetry

# Composite Higgs in strongly coupled systems:

SU($N_{color}$ ≥2 ) gauge fields + $N_{flavor}$ fermions in some representation



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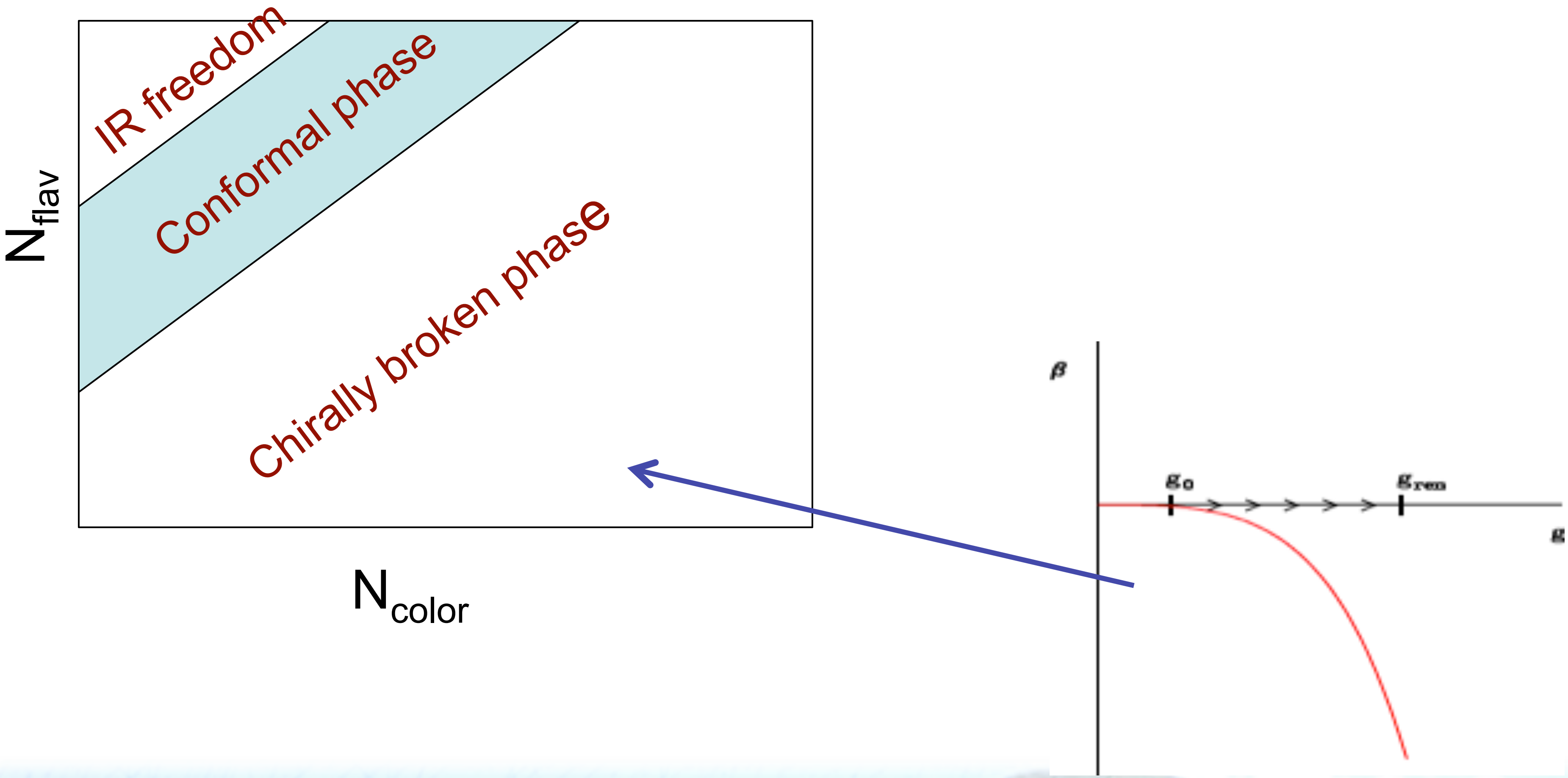

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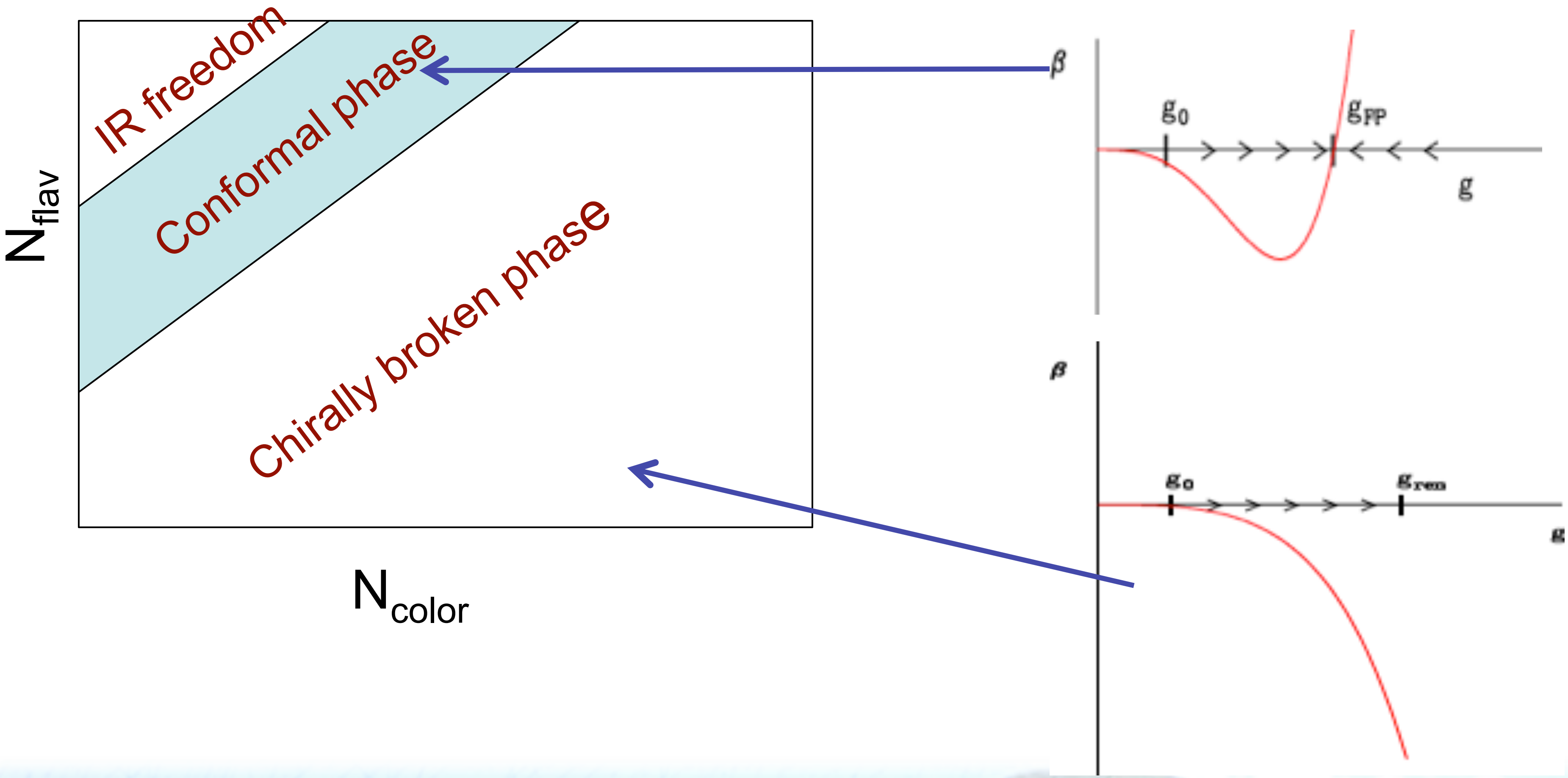

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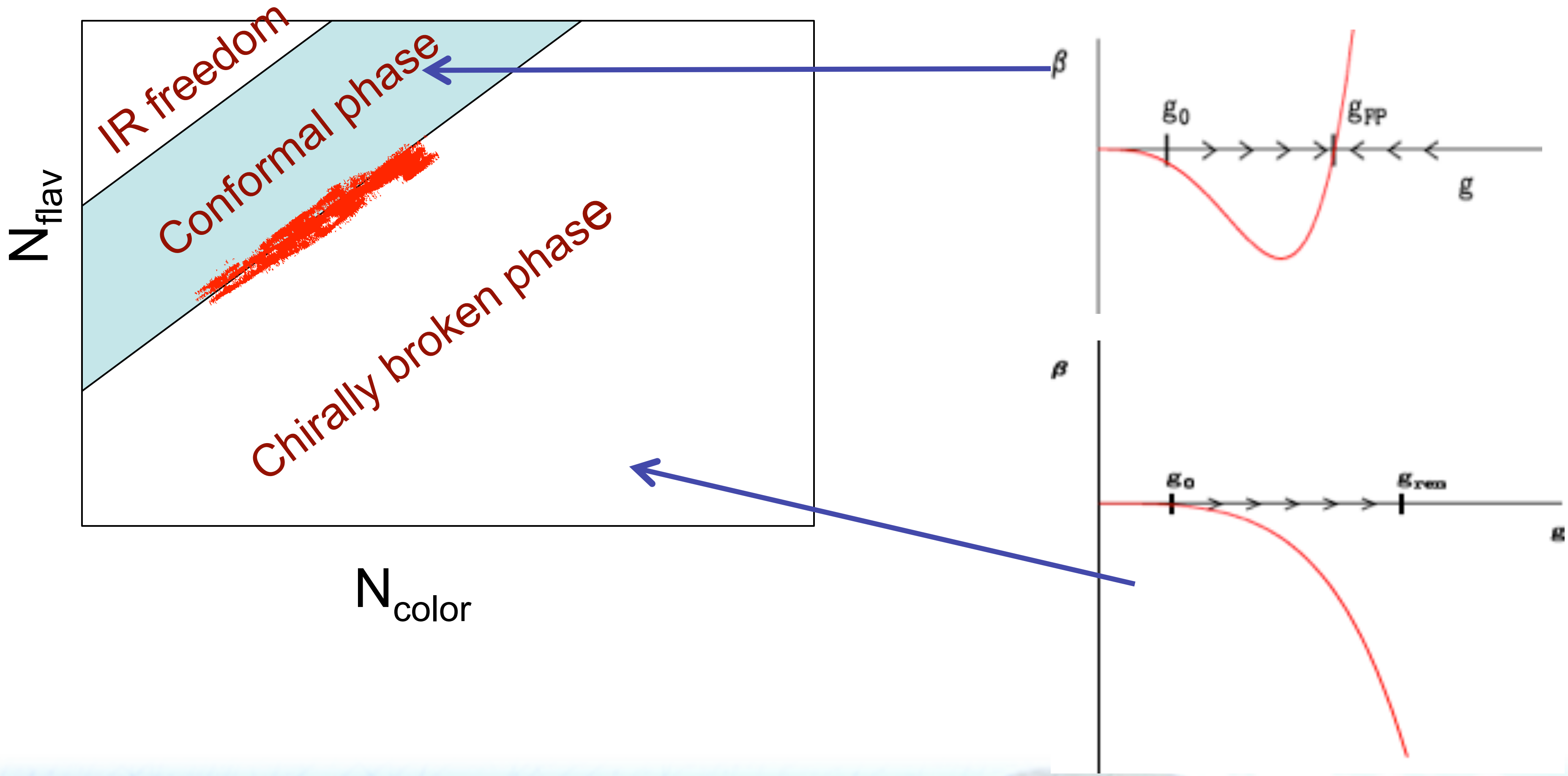

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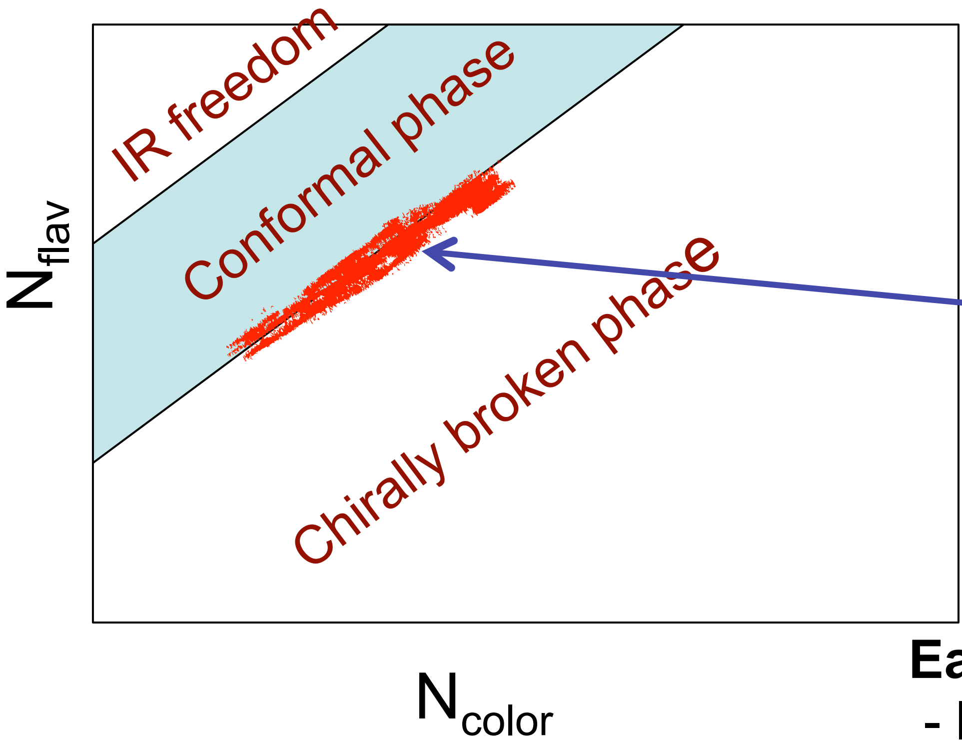
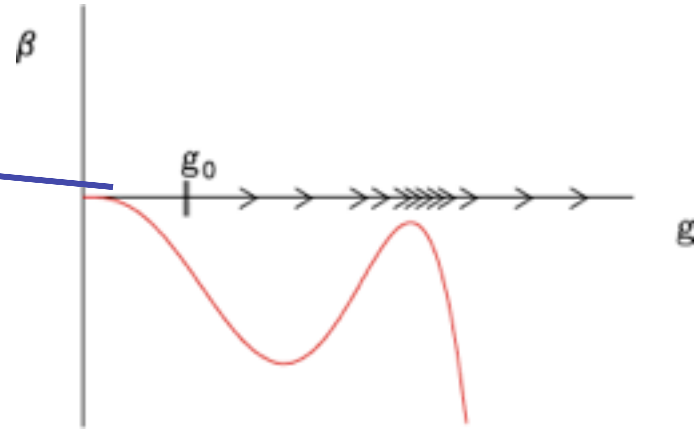


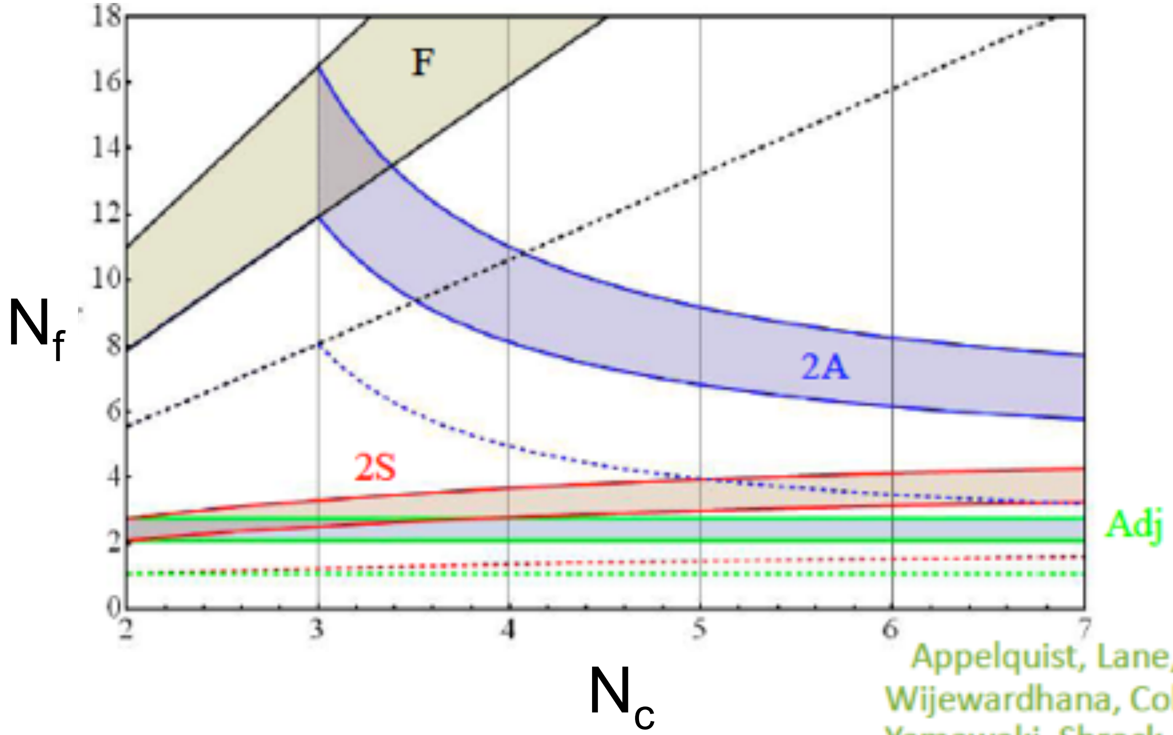




**Early** lattice results **suggest**

- light scalar
- enhanced chiral condensate
- suppressed S parameter

# Roadmap for the conformal window

S-D type calculations




Shaded: conformal
Below : confining
Above: IR free
Dotted lines: 2-loop PT

fermion representation:
Fundamental
Adjoint
2Symmetric
2Antisymm

Appelquist, Lane, Mahanta, Wijewardhana, Cohen, Georgi, Yamawaki, Shrock, Dietrich, Sannino, Tuominen

Needs non-perturbative verification!

# ~~Roadmap~~ for the conformal window
# Cartoon

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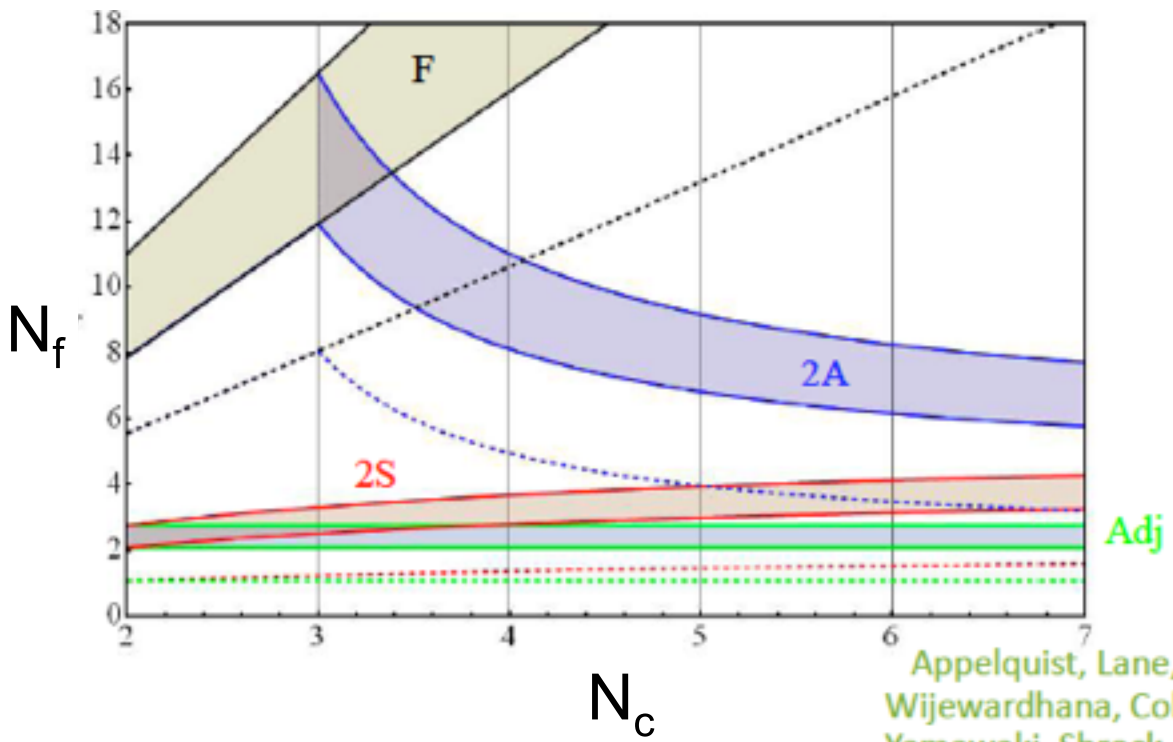


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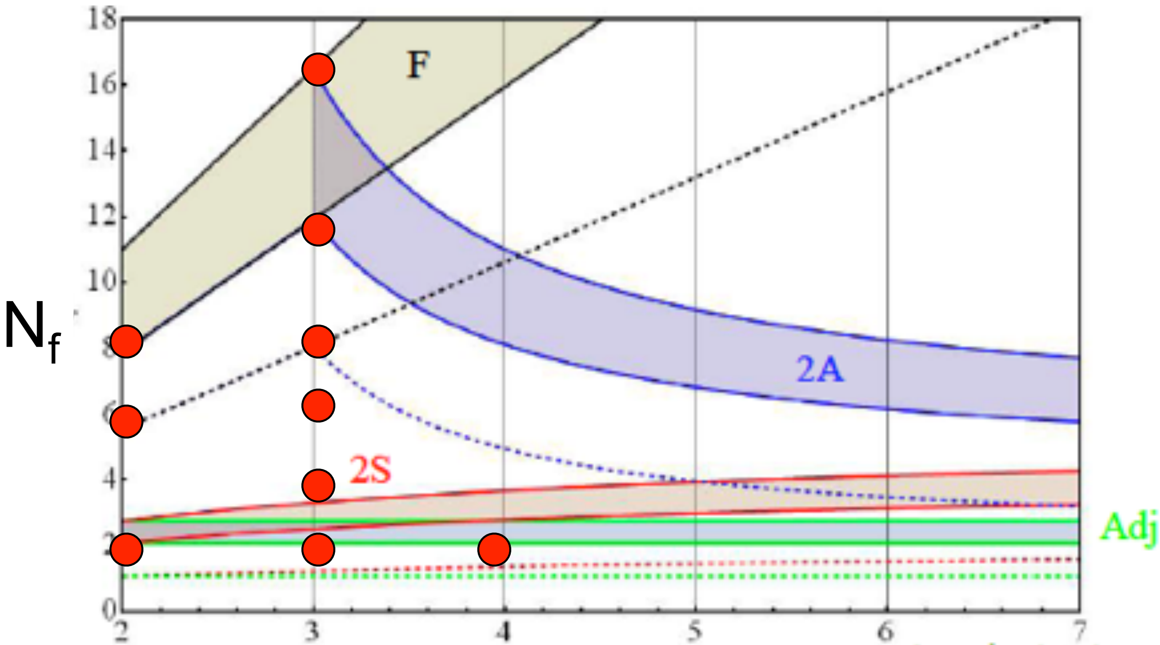


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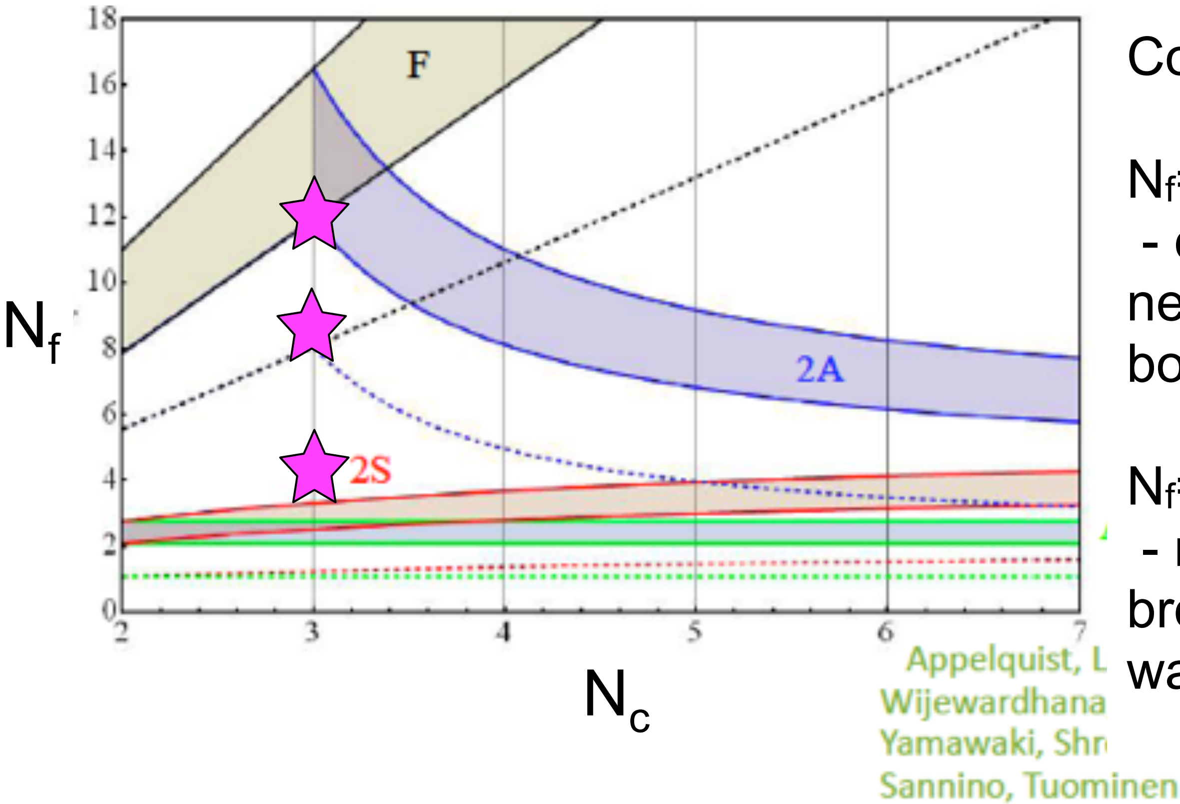
# In this talk: $N_f$ = 4, 8 and 12 fundamental fermions




Concentrate on

$N_f$=12:
- controversial system near the conformal boundary

$N_f$=8:
- most likely chirally broken but could be walking

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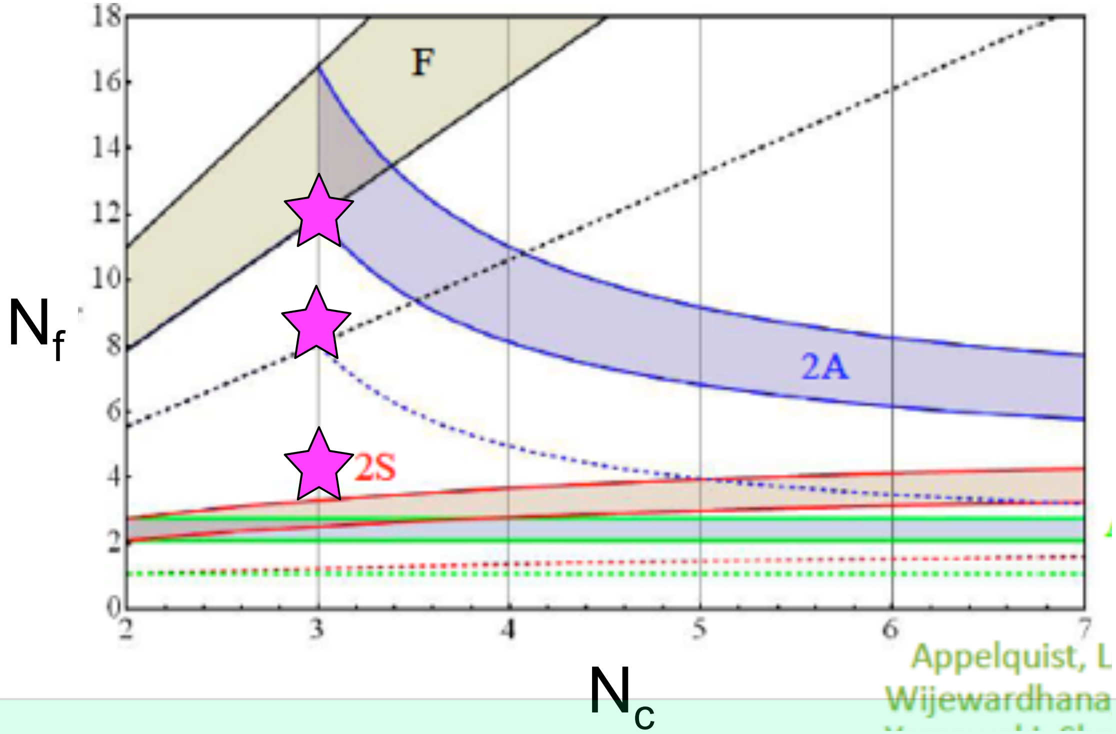


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$N_f$=8:
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Questions to answer:
- Is the system conformal or chirally broken (and walking)?
- Is there a light scalar?
- Is the S parameter small? What is the anomalous mass dim.?
- .......

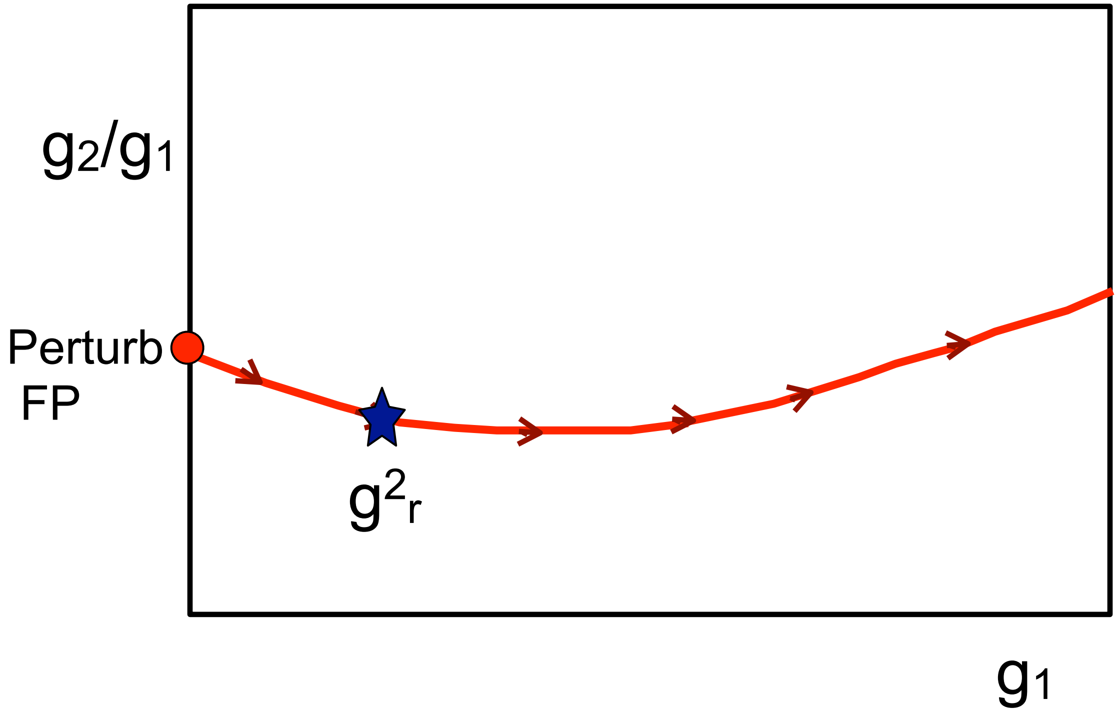
Simple enough .... cannot be much harder than QCD

It is surprisingly difficult to distinguish conformal, walking, and chirally broken systems on the lattice

# Fixed point structure of a **chirally broken system**

Wilson RG

m=0 critical surface: **one fixed point**

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g=0,m=0 : 2 relevant directions

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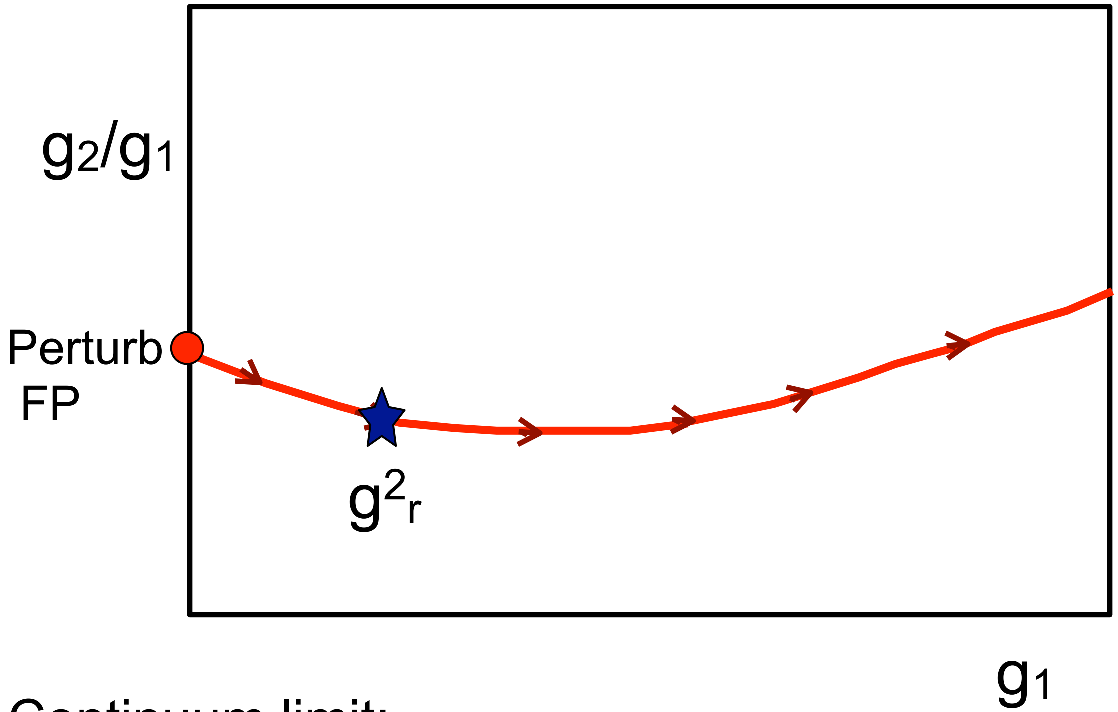


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Continuum limit:

Tune bare $g^2 \to 0$ and $m \to 0$ : renormalized $g^2$ anywhere on renormalized trajectory

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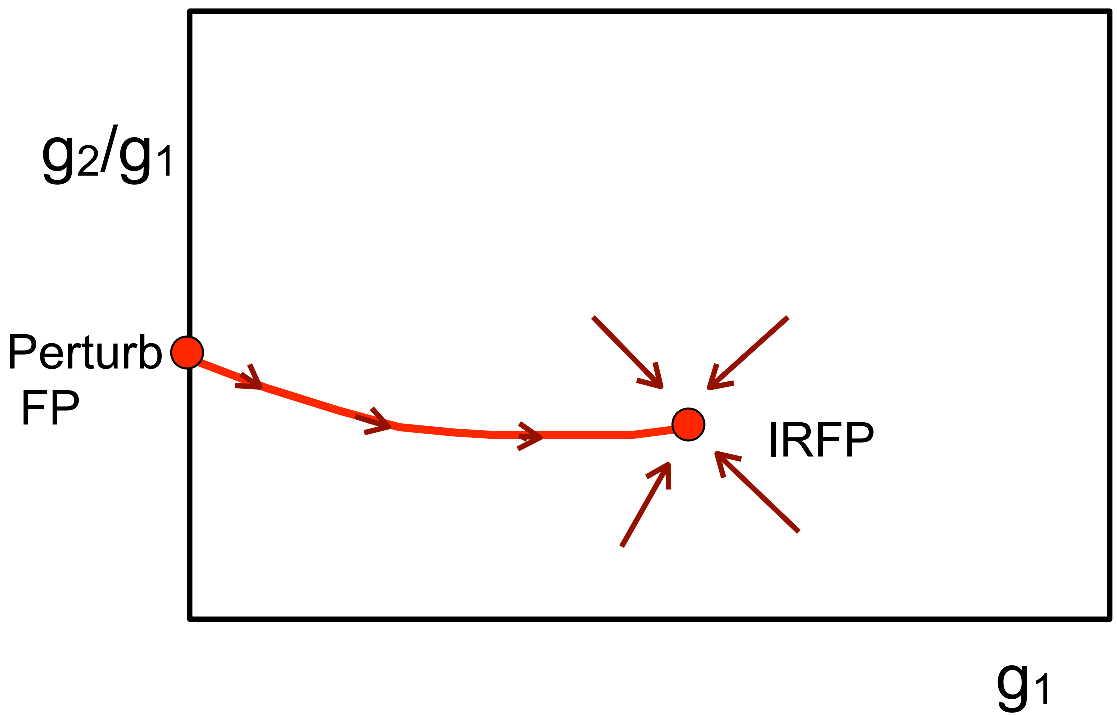


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$g=g_{IRFP}$,m=0 : 1 relevant direction

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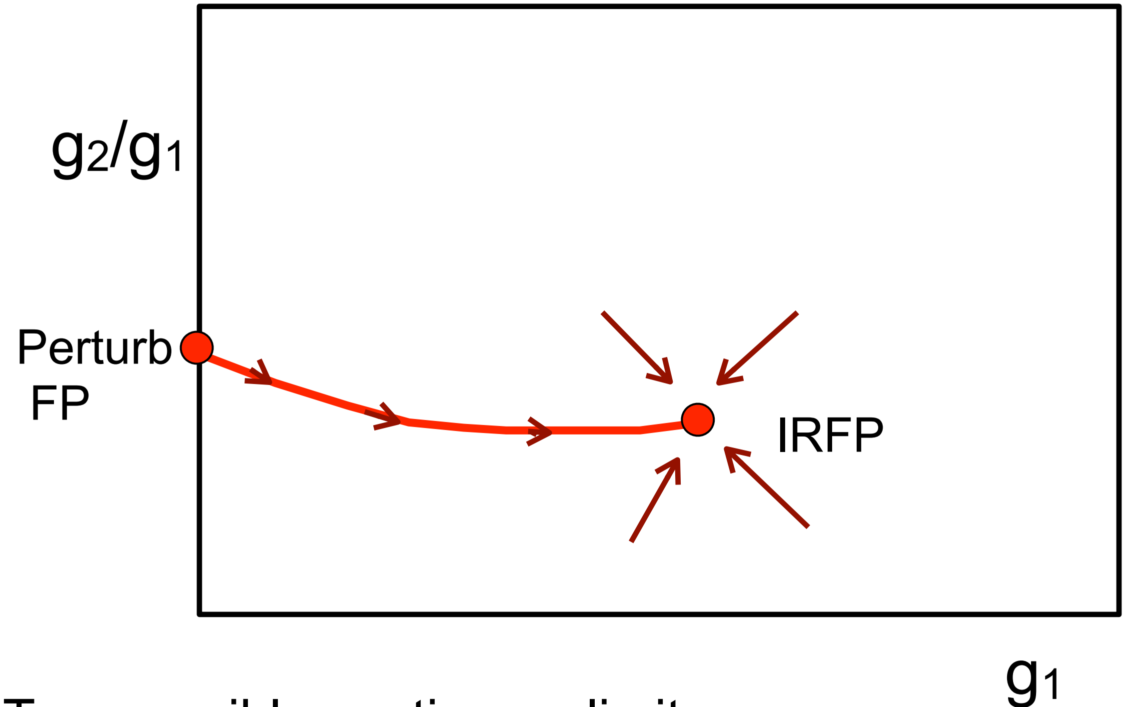


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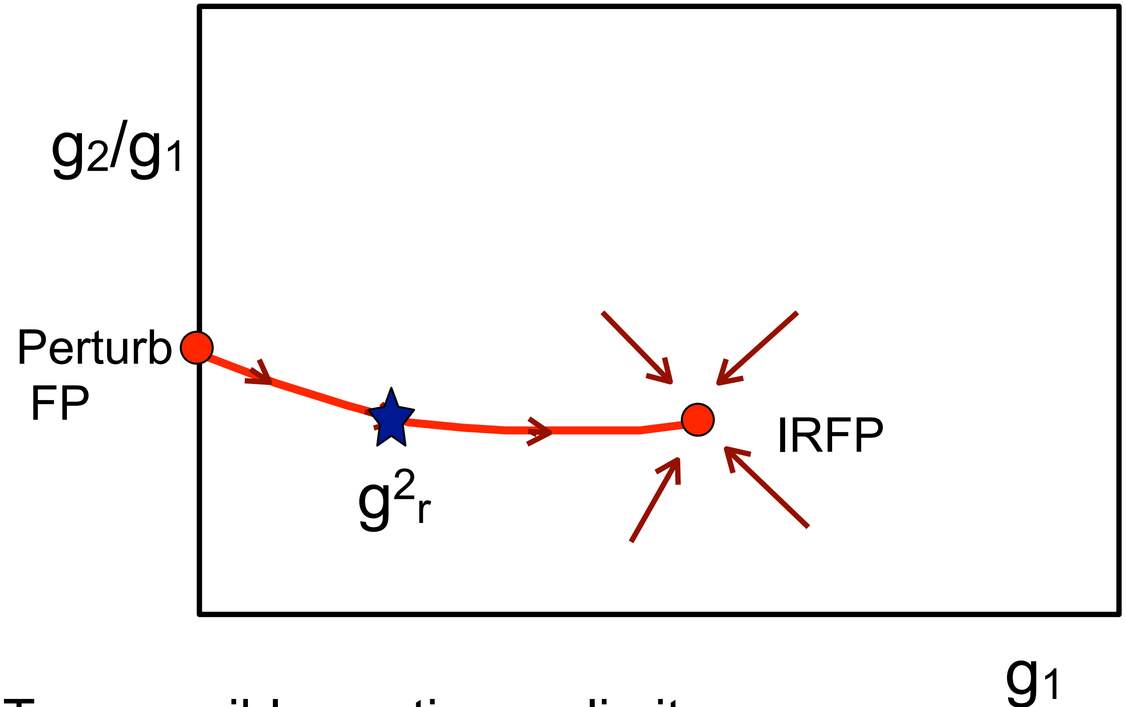


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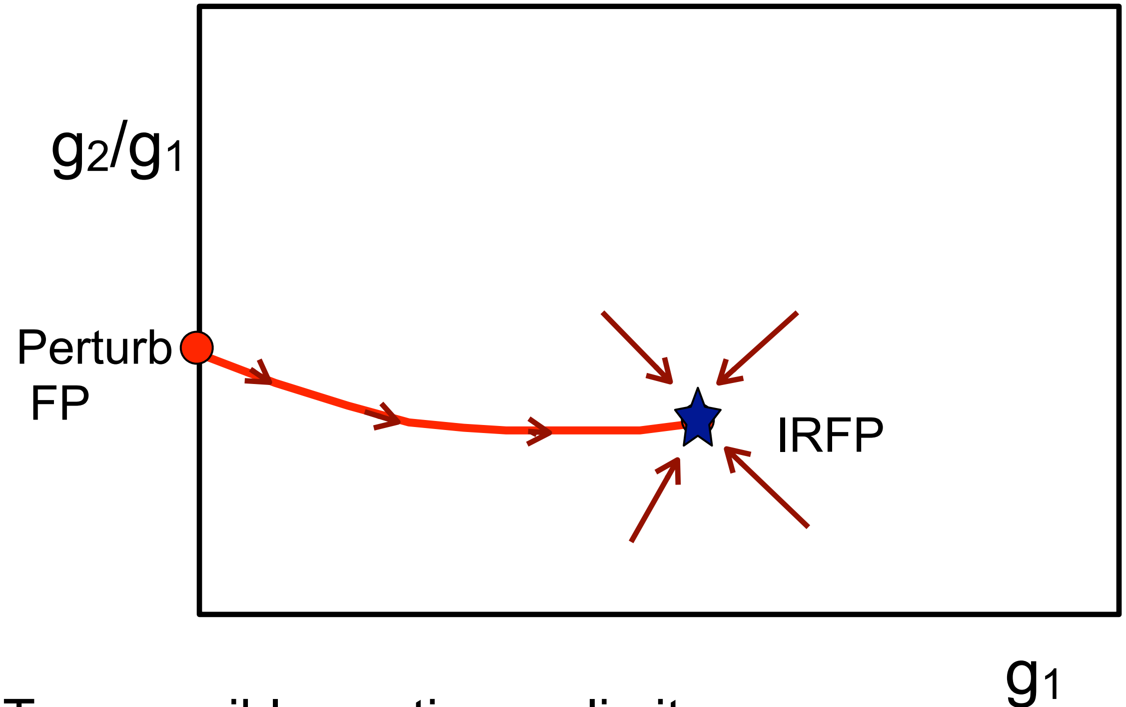


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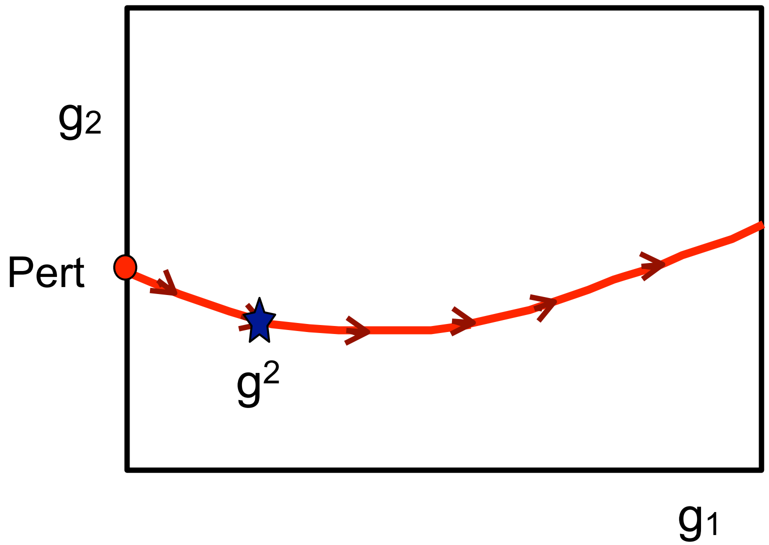


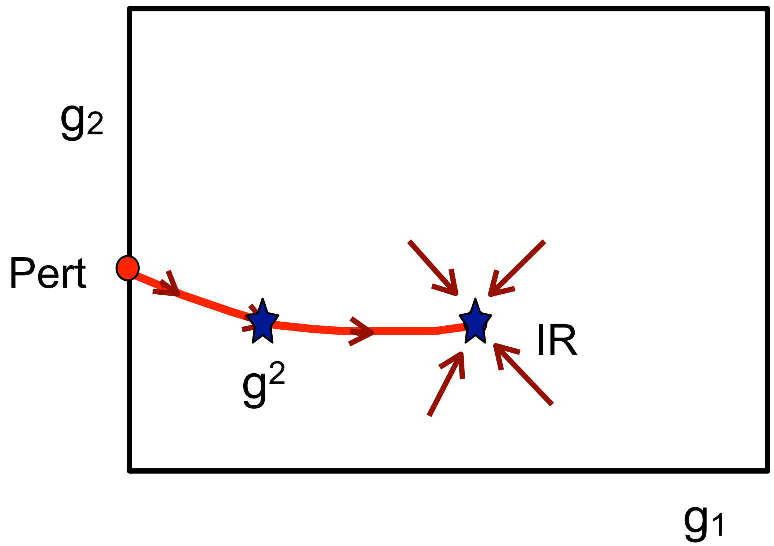


- they look very similar along the RT
- if the gauge coupling "walks" : g is nearly **marginal !** **(non-QCD like)**

# SU(3) gauge with $N_f$= 4, 8 and 12 fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density $\rho(\lambda)$
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
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Mostly $N_f$=12 flavor to test the methods and understand/resolve existing controversies.

# Dirac operator eigenvalue spectrum and spectral density

Chirally broken systems: $\rho(0)=\Sigma/\pi$

Conformal systems are chirally symmetric: $\rho(0)=0$

critical behavior: spectral density scales as $\rho(\lambda)\propto\lambda^{\alpha}$ , λ≈0

.

The **mode number** $\nu(\lambda)=V\int_{-\lambda}^{\lambda}\rho(\omega)d\omega\propto V\lambda^{\alpha+1}$ is RG invariant

(Giusti,Luscher)

i.e. unchanged under scale change s: $V\to s^4 V$, $\lambda\to\lambda/s^{1+\gamma}$ , $\nu\to\nu$

→ α is related to the anomalous dimension (Zwicky,DelDebbio;Patella)

$$\frac{4}{1+\alpha}=y_m=1+\gamma_m$$

# Scaling of the Dirac eigenvalue spectrum - **conformal system**

Eigenvalue density scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

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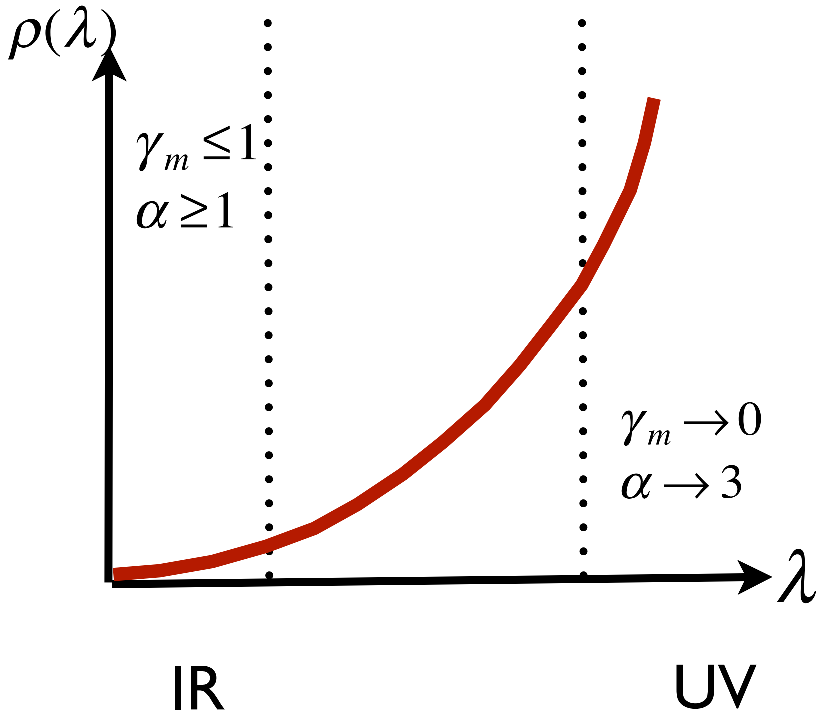


**IR** – small λ region:

$$\gamma_m(\lambda \to 0) = \gamma_m^*$$

predicts the universal anomalous dimension at the IRFP

**UV** – large λ =O(1) region:
if governed by the asymptotically free perturbative FP

$$\gamma_m(\lambda = \mathcal{O}(1)) = \gamma_0 g^2 + \ldots$$

**In between:**
scale dependent effective $\gamma_m$

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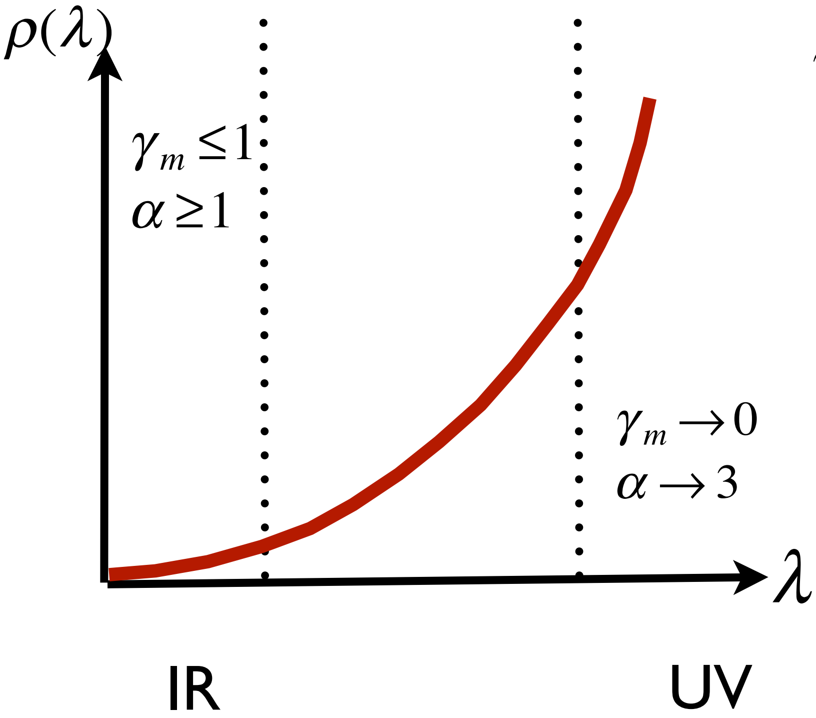


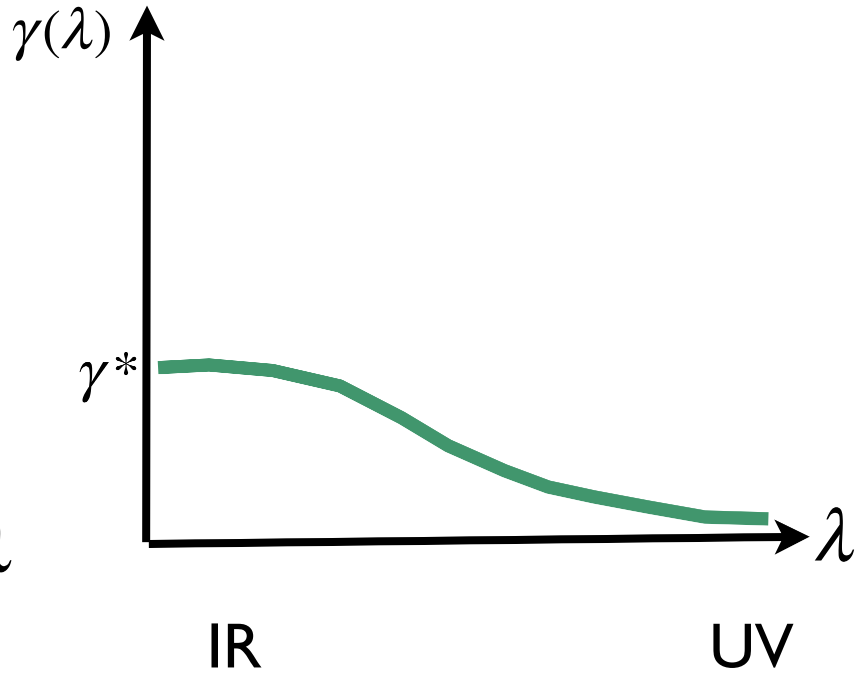

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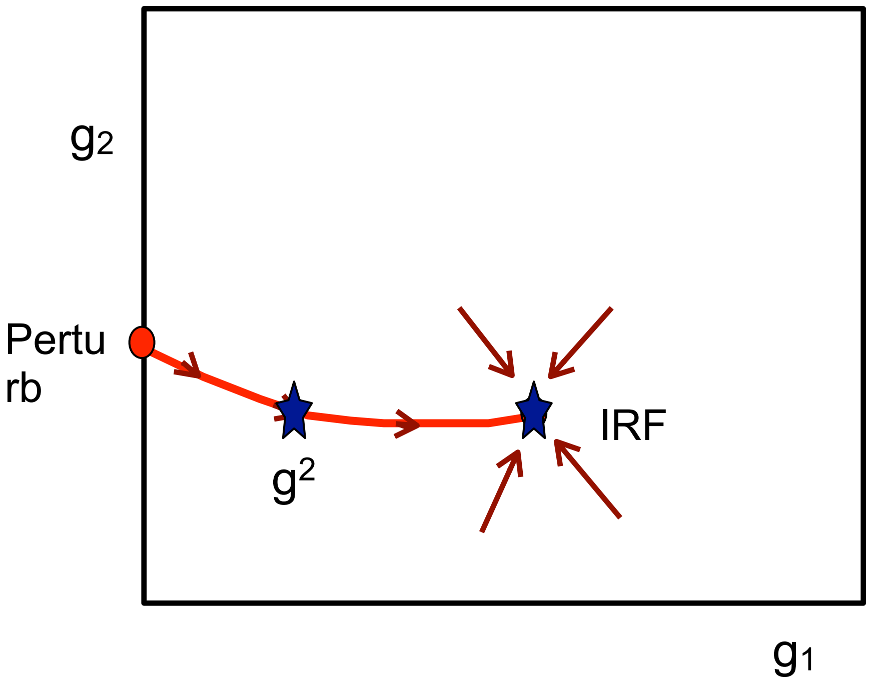


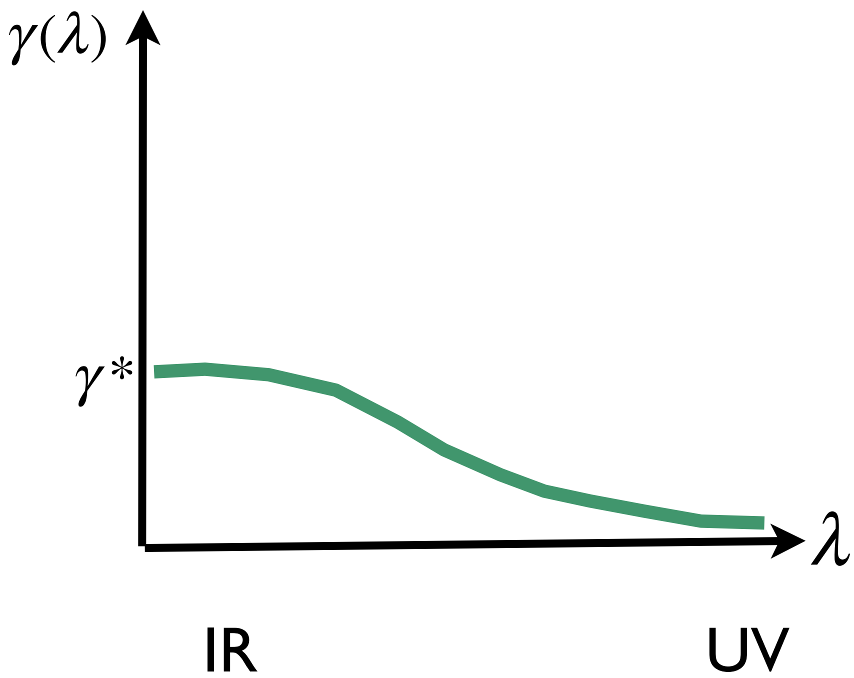

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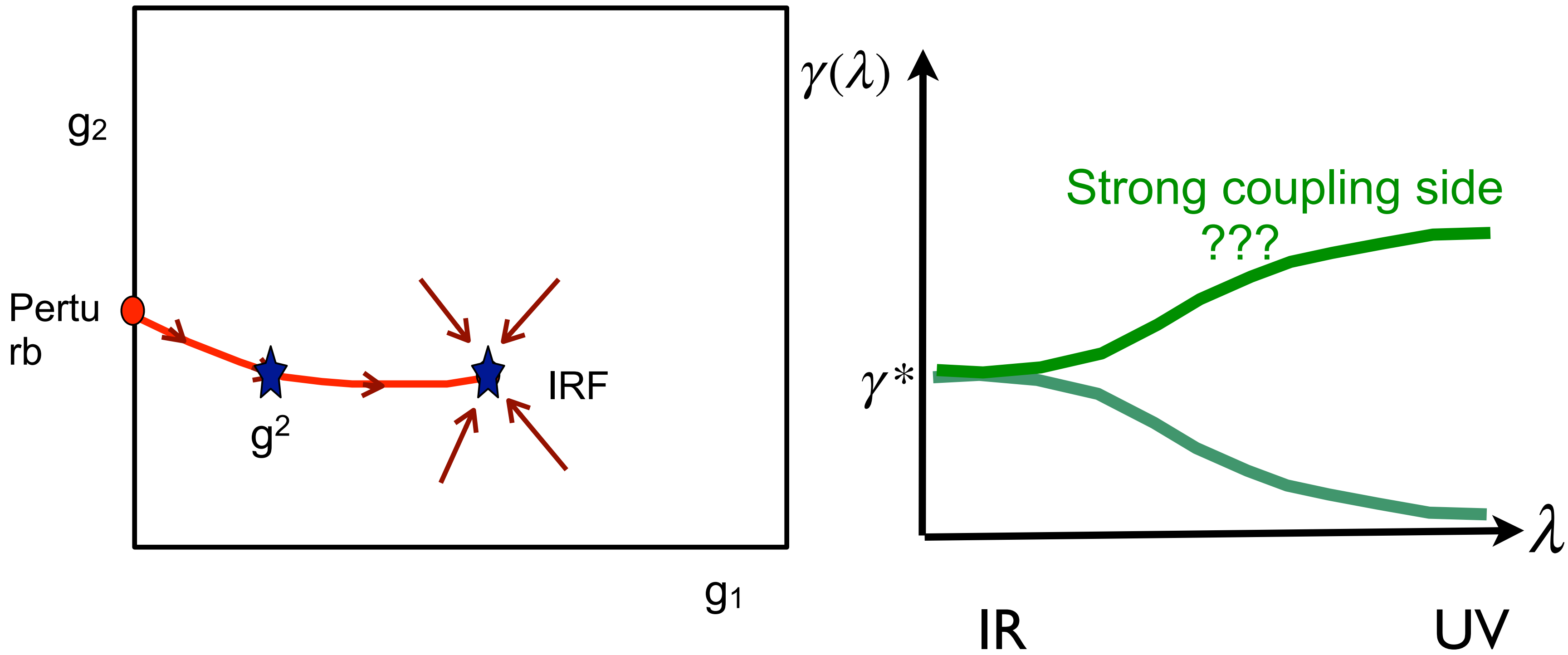

# Scaling of the Dirac eigenvalue spectrum - chirally broken system

The picture is still valid in the UV and moderate energy range

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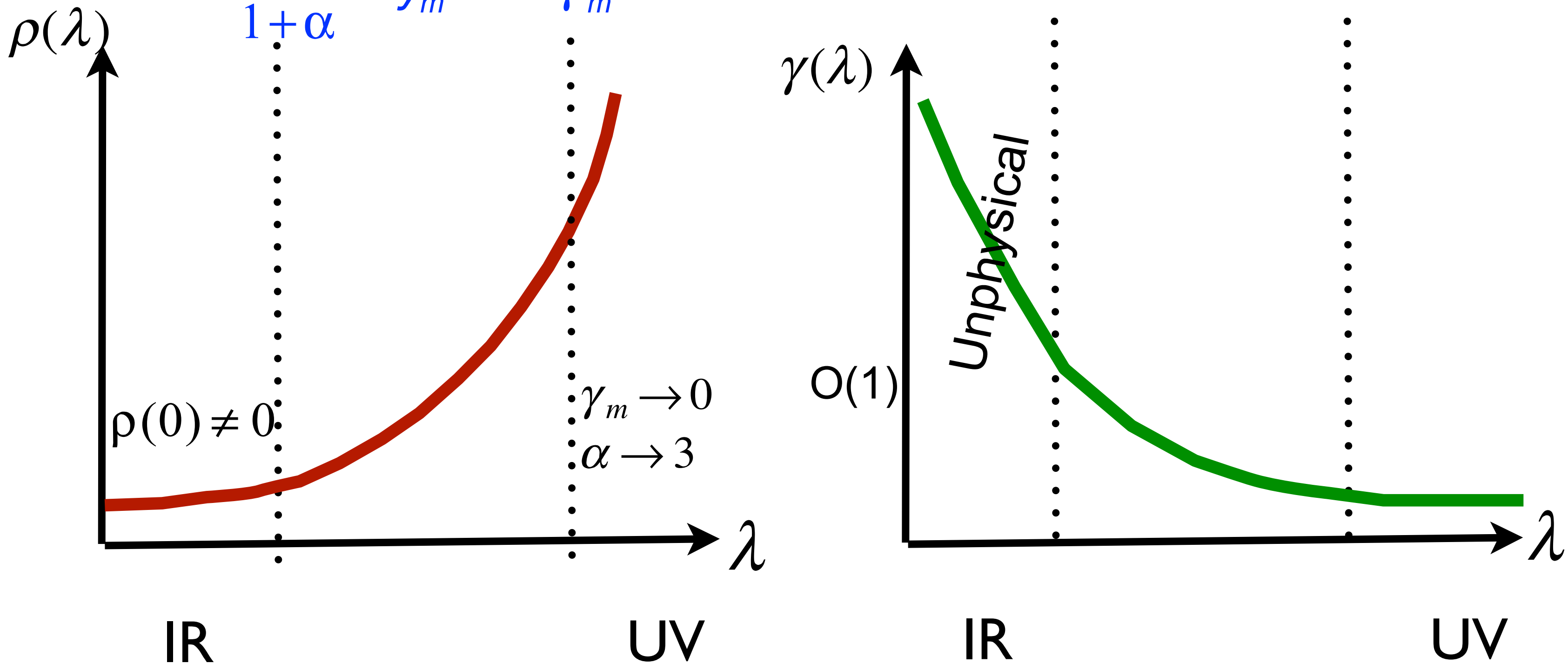

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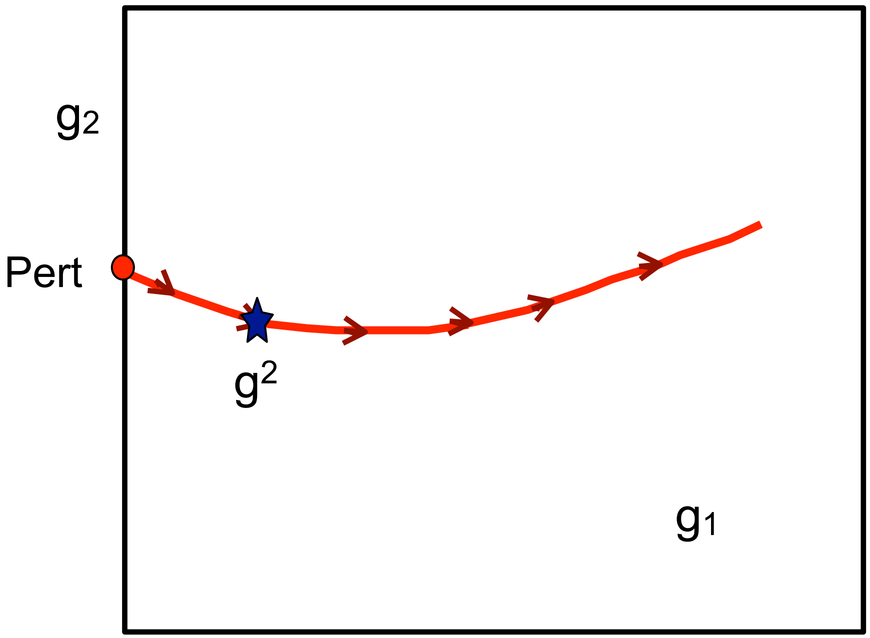


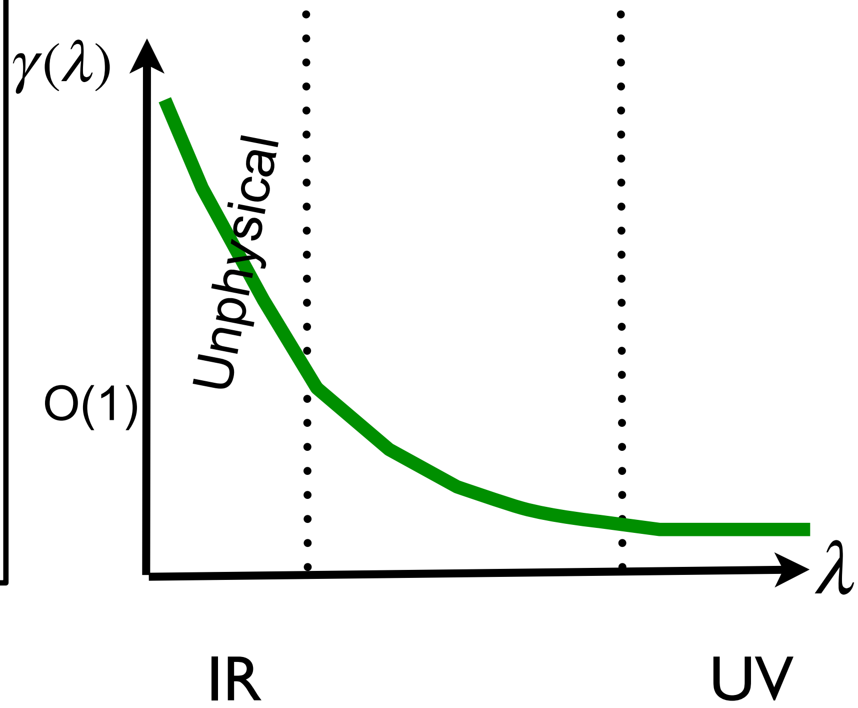

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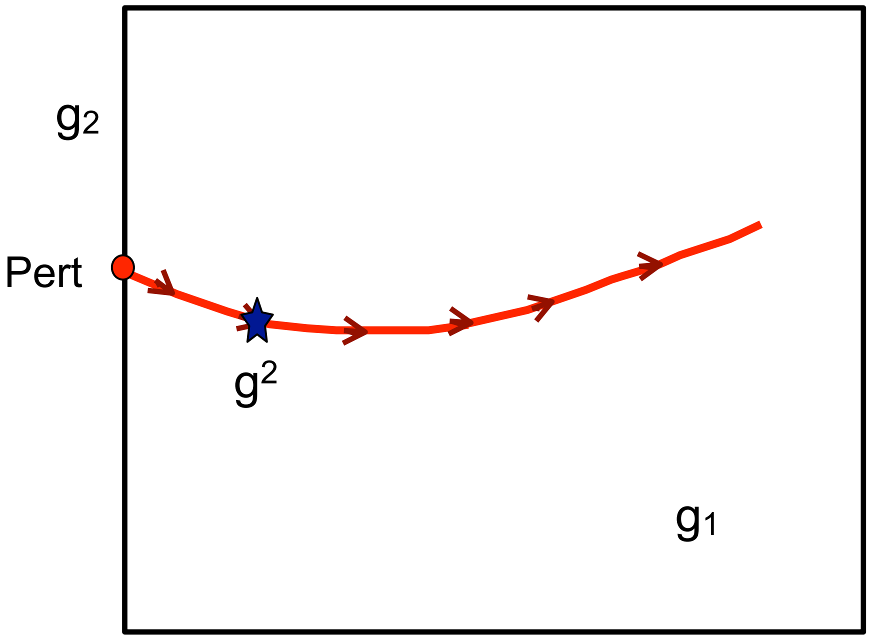
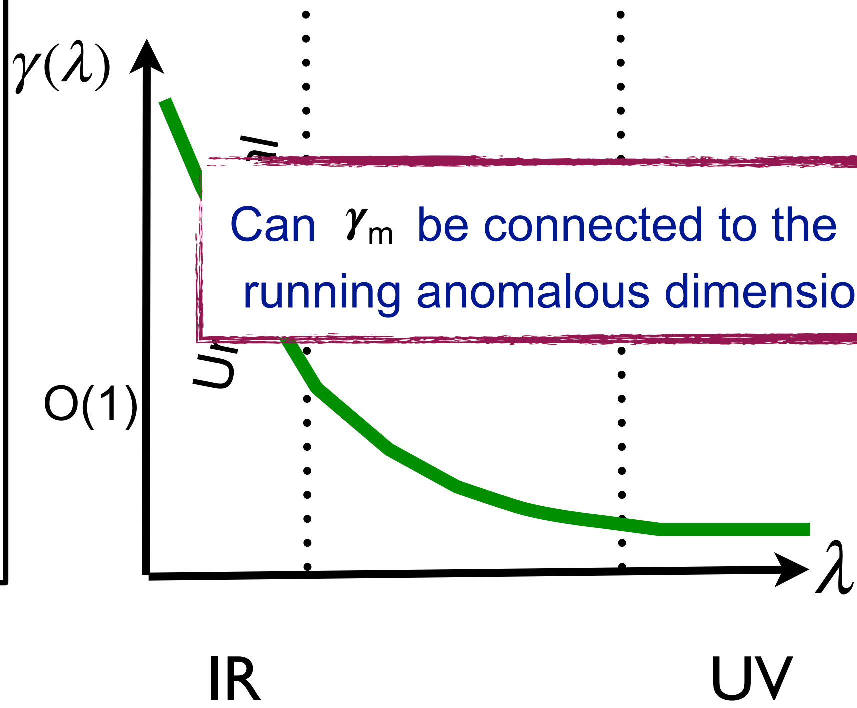



Can $\gamma_m$ be connected to the running anomalous dimension?

# Dirac operator eigenvalue spectrum and spectral density

Goal:

- calculate $\nu(\lambda)$ stochastically
- fit $\nu(\lambda) \propto \lambda^{\alpha}$ in small λ ranges
- extract the scale dependent $\gamma_m(\lambda)$

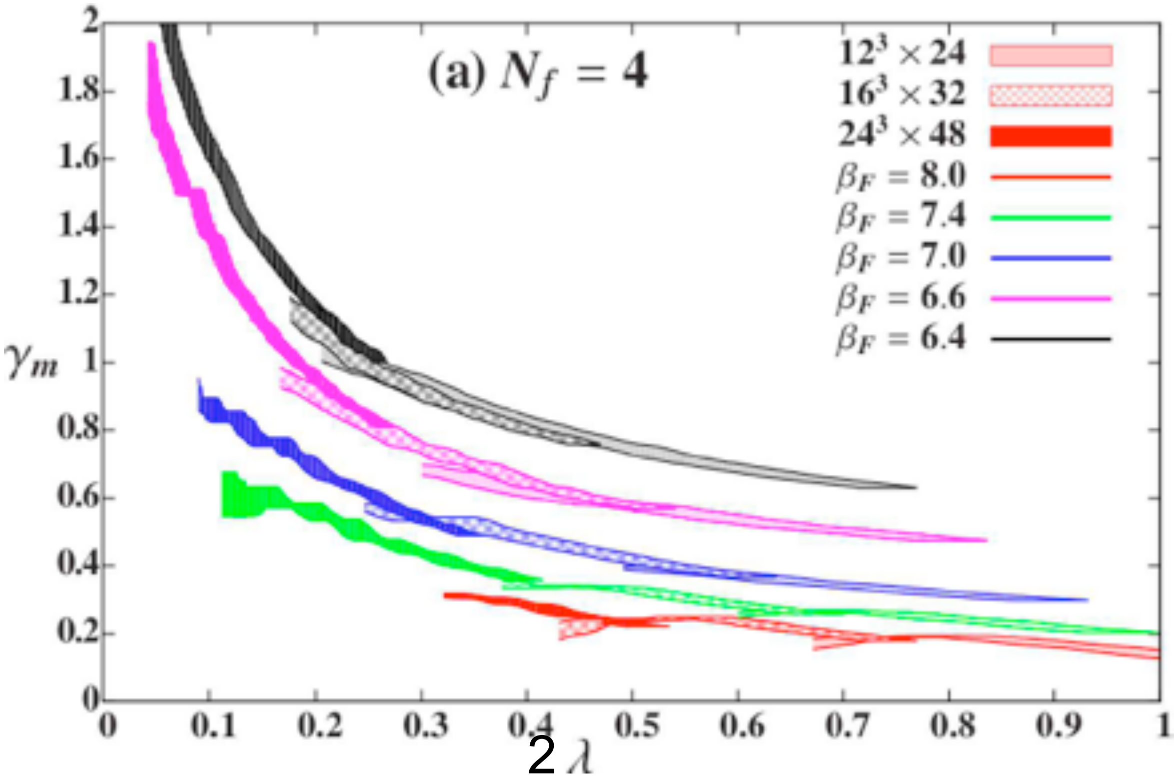
.

This should be done in the chiral m=0 infinite volume L→∞ limit :

finite mass, volume introduces only small λ transient effects

# Results: $N_f$ =4

Broken chiral symmetry in IR, asymptotic freedom in UV




Lattice spacing from Wilson flow:

$$a_{6.4} \ / \ a_{7.4} = 2.84(3)$$

$$a_{6.6} \ / \ a_{7.4} = 2.20(5)$$

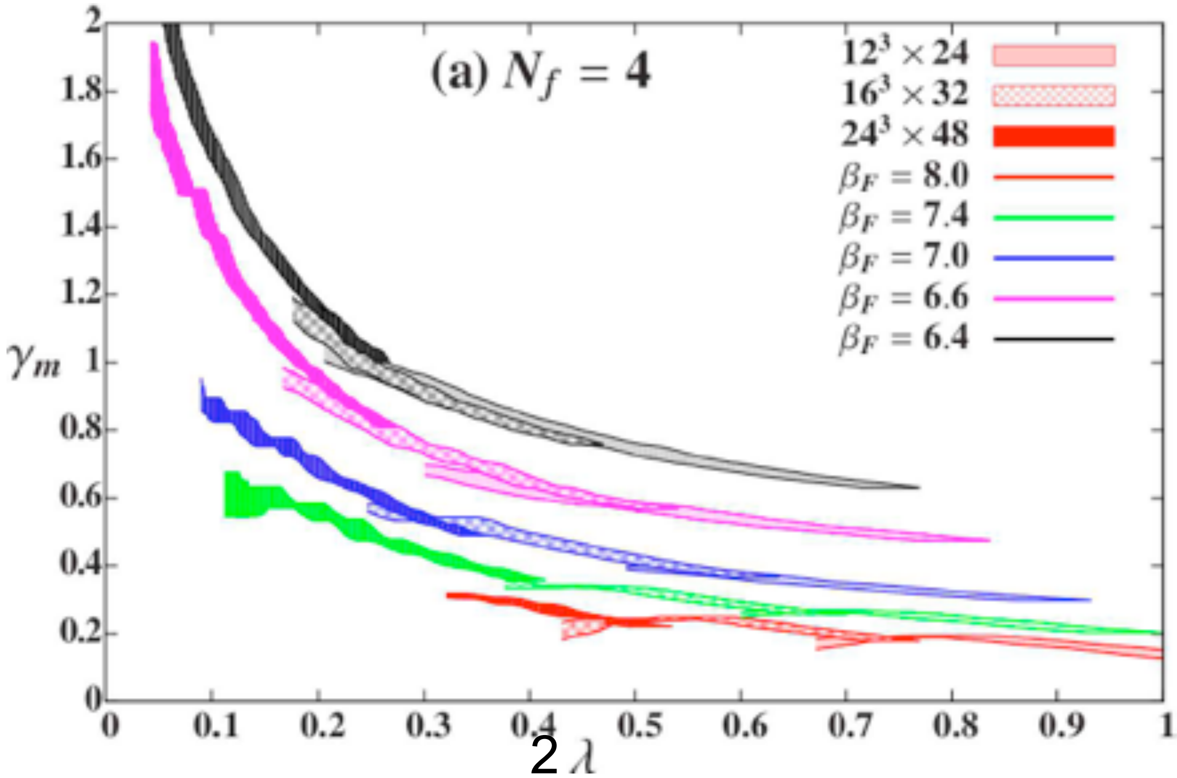
$$a_{7.0} \ / \ a_{7.4} = 1.45(3)$$

$$a_{8.0} \ / \ a_{7.4} = 0.60(4)$$

# Rescaling: $N_f = 4$

The dimension of λ is carried by the lattice spacing: $\lambda_{lat} = \lambda_p a$
Rescale to a common physical scale:

$$\lambda_\beta \rightarrow \lambda_\beta \left( \frac{a_{7.4}}{a_\beta} \right)^{1+\gamma_m(\lambda_\beta)}$$

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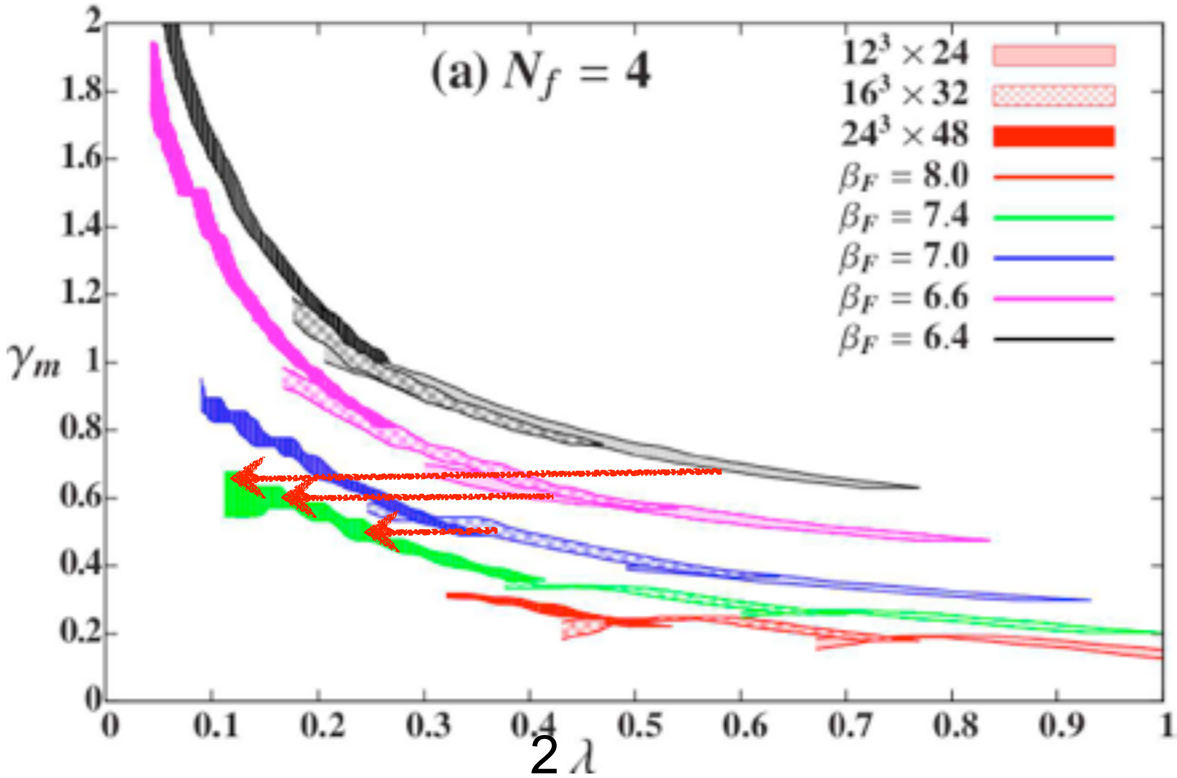


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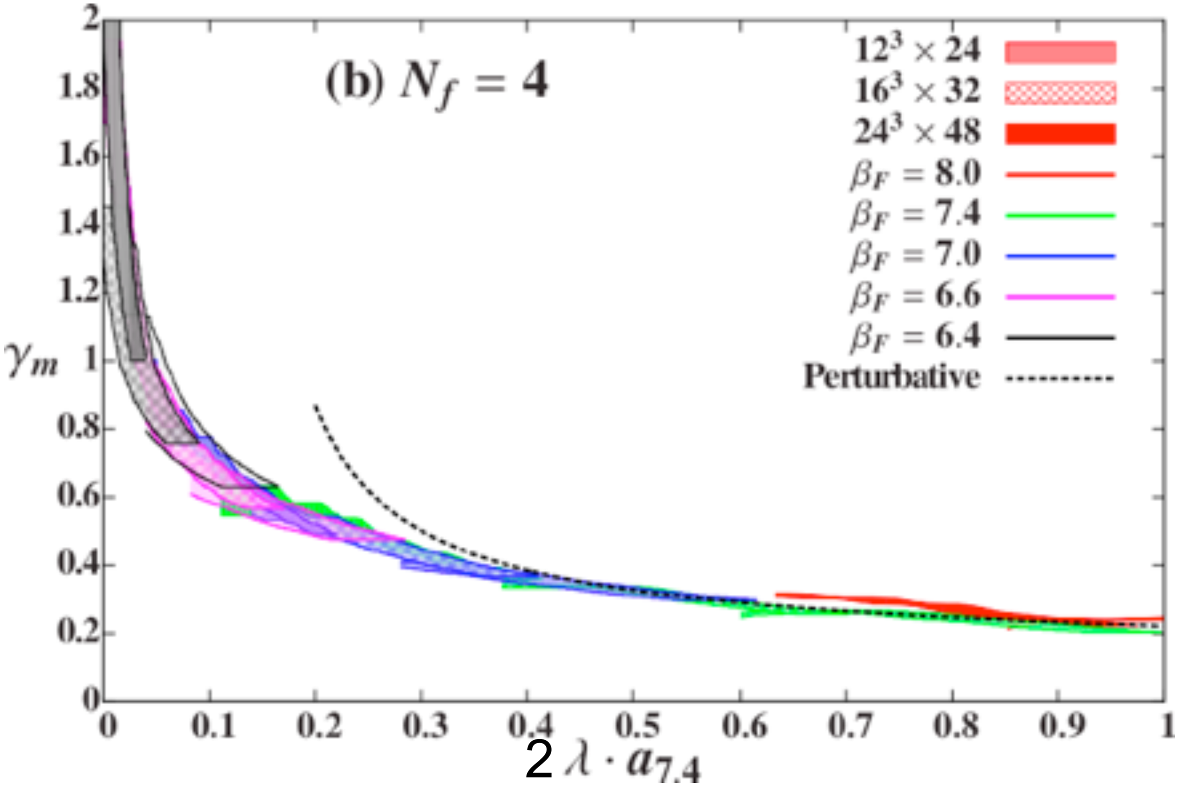



Universal curve covering almost 2 orders of magnitude in energy!

Perturbative: functional form from 1-loop PT, relative scale is fitted

Most of these data were obtained on deconfined (small) volumes at m=0!

# Results: $N_f$ =12

Controversial system :

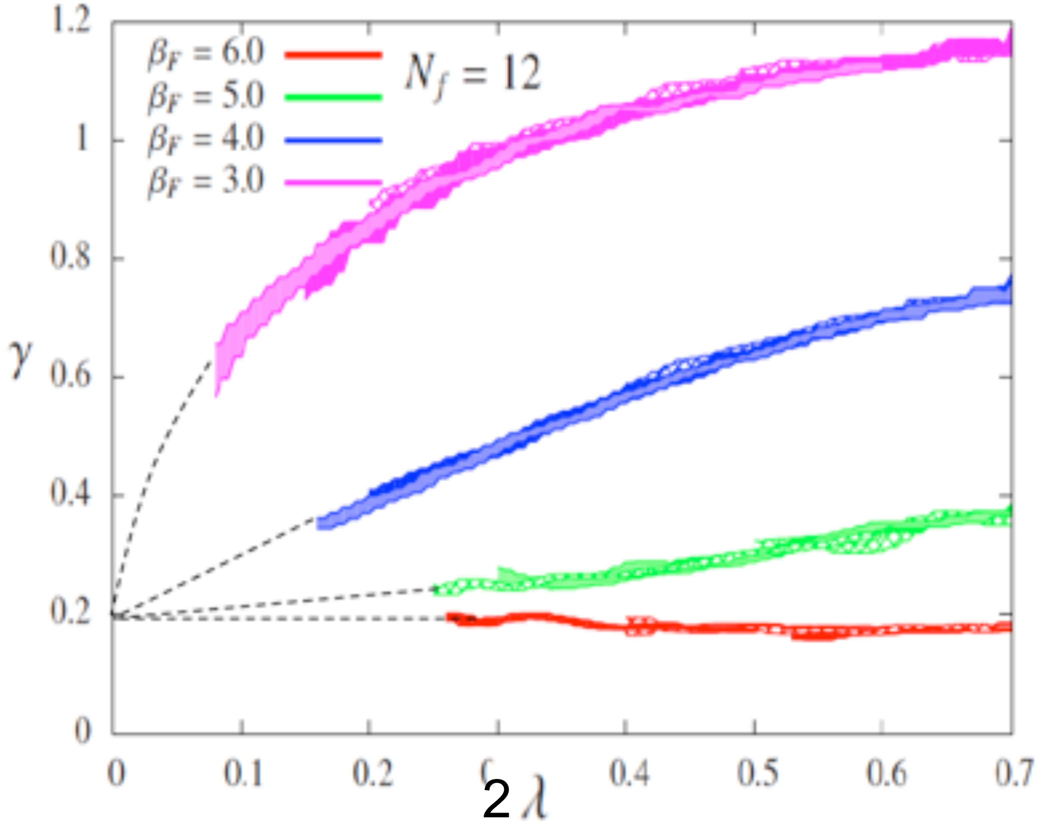
SD predicts it is right at the conformal boundary

early studies suggested it is conformal

.... then chirally broken ... then conformal ....

The model could be phenomenologically unimportant, but it is a great model to test methods / understanding!

# Spectral density results: $N_f$ =12




β=3.0, 4.0, 5.0, 6.0

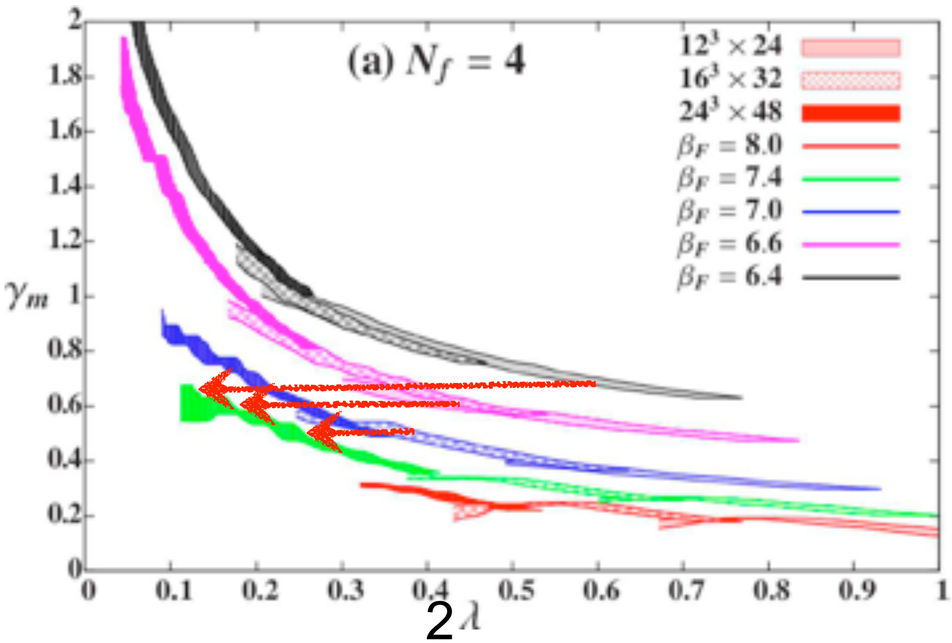
- There is no sign of asymptotic freedom behavior for β<6.0,

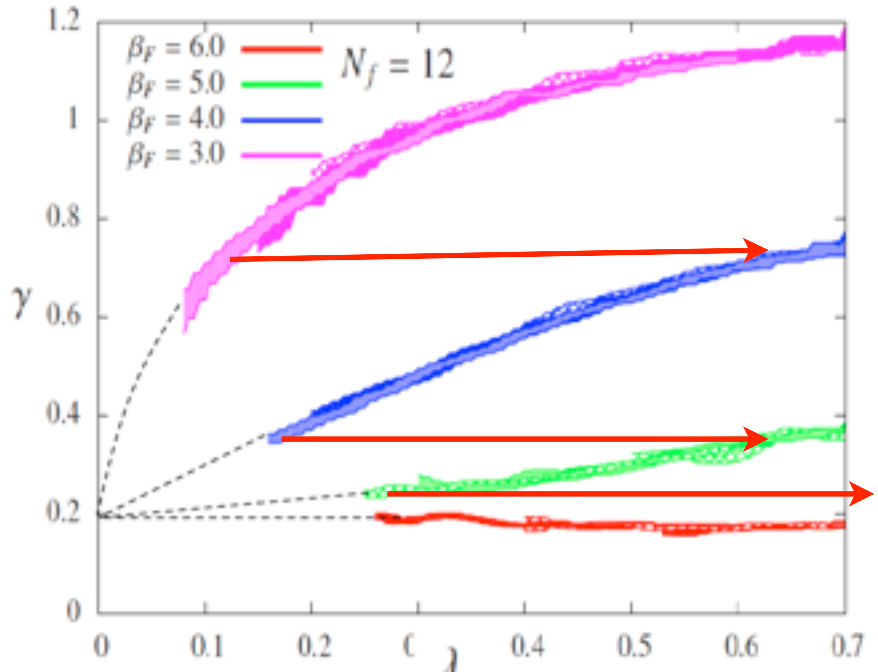
  $\gamma_m$ grows towards UV

- Not possible to rescale different β's to a single universal curve

Looks as if there was an IRFP between β=5.0 -6.0

# Rescaling $N_f$=4 vs $N_f$=12

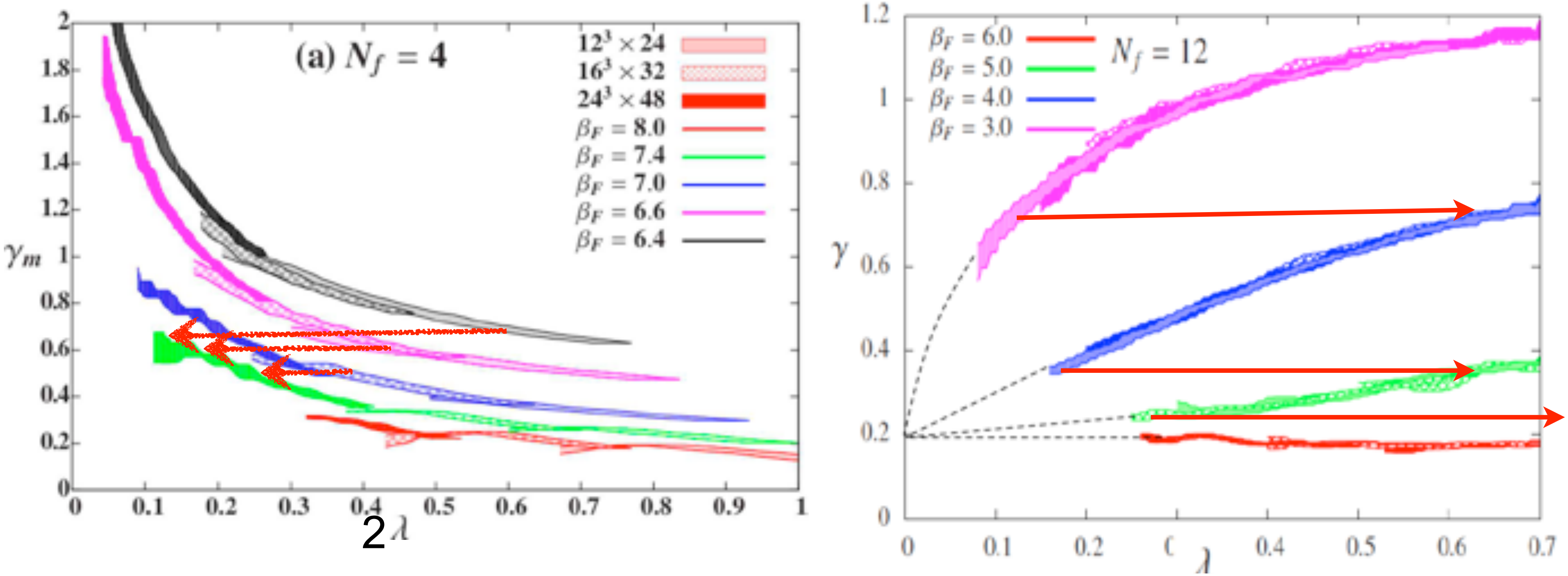



$N_f$=4 : smaller β matches to the left (forward flow)

$N_f$=12 : no consistent rescaling but even an approximate one matches to the right of β<6.0

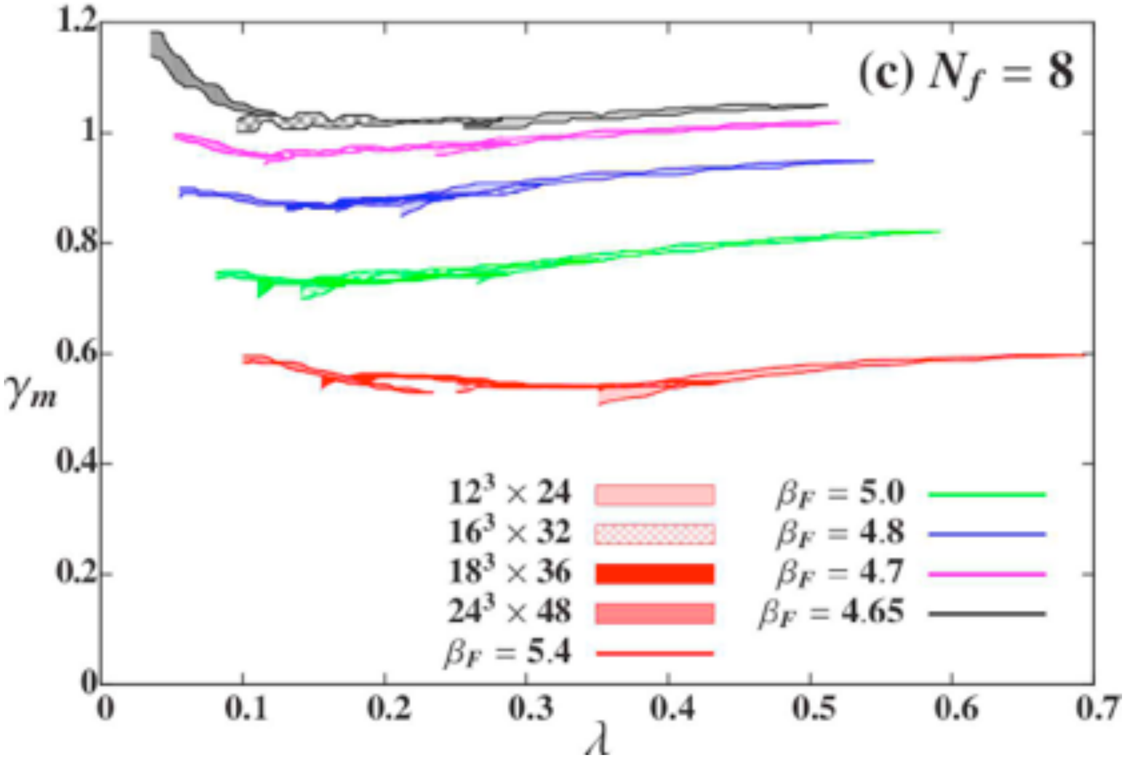
# Rescaling $N_f$=4 vs $N_f$=12




Spectral density appears to be a very sensitive test to identify a conformal system

# Anomalous dimension, $N_f$ =8

Expected to be chirally broken - looks like walking!




-No asymptotic free scaling
-No rescale of different couplings

-When $\gamma_m \sim 1$ in the UV, the $S^4b$ phase develops

# Dirac operator eigenvalue spectrum and spectral density

Unique & promising method !

- Can distinguish strong and weak coupling region of conformal /chirally broken systems

Predictions:

$N_f$=4 : scaling & anomalous dimension

$N_f$=12 : looks conformal

$N_f$=8 : could be walking with large anomalous dimension!

# II : Finite size scaling

# II : Finite size scaling

HISTORY:

Several groups attempted finite size scaling for $N_f$=12

- curve collapse is possible but the predicted scaling exponent is strongly operator dependent

CONCLUSION 1:

No consistent finite size scaling suggests that the system is not conformal

CONCLUSION 2:

Problems are due to the near-marginal gauge coupling. Take this into account and things become consistent

# Finite size scaling

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

$g_i$ with scaling dimensions $y_i < 0$.

Renormalization group arguments in volume $L^3$ predict scaling of physical masses as

$$M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $m \to 0, \quad L \to \infty : \quad g_i m^{-y_i/y_0} \to 0$

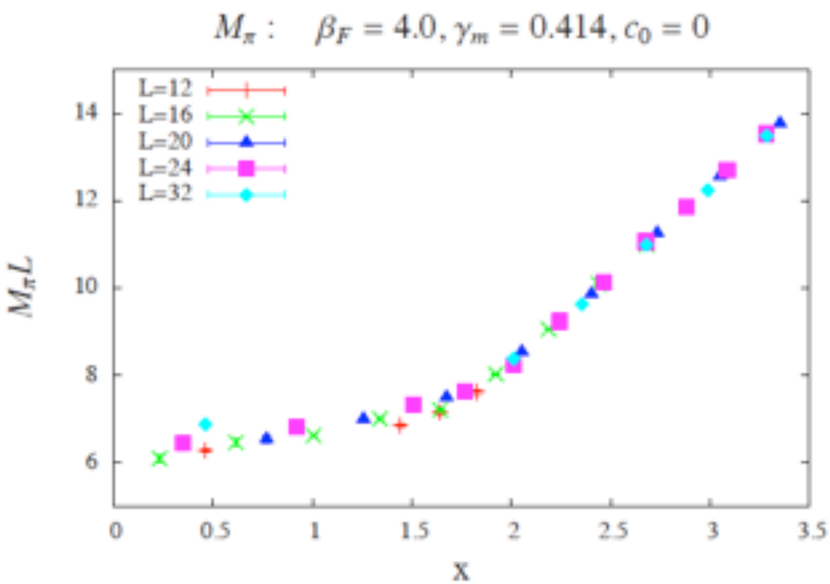
$$M_H L = f(x), \quad x = Lm^{1/y_m}$$

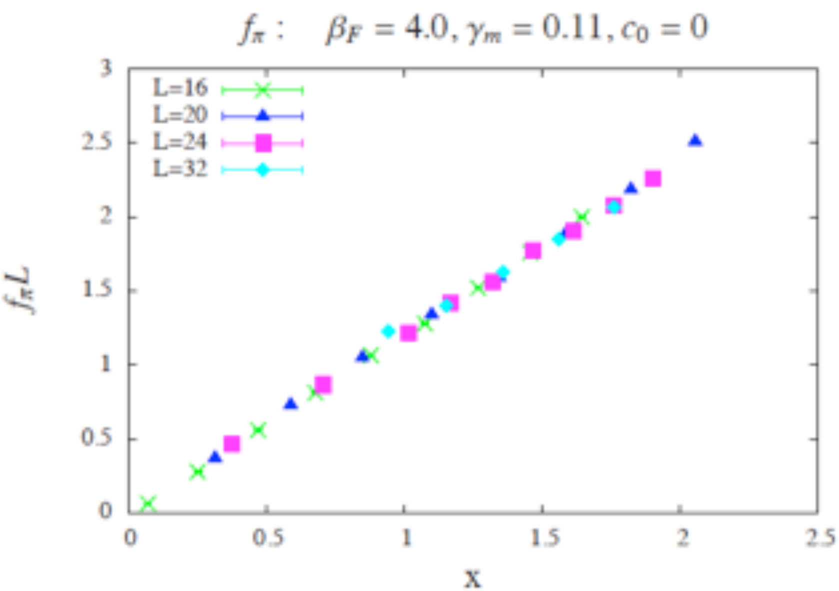
–tune $y_m$ until different volumes "collapse"

# Finite size scaling with nHYP action, $N_f$=12

$\beta$= 4.0 (meson spectrum matches LHC $\beta$=2.2 closely)

- good curve collapse for larger $x = Lm^{1/y_m}$
- inconsistent exponents (See results from LHC, KMI as well)




$M_\pi$ : $y_m$=1.408(10)

$f_\pi$ : $y_m$=1.11(5)

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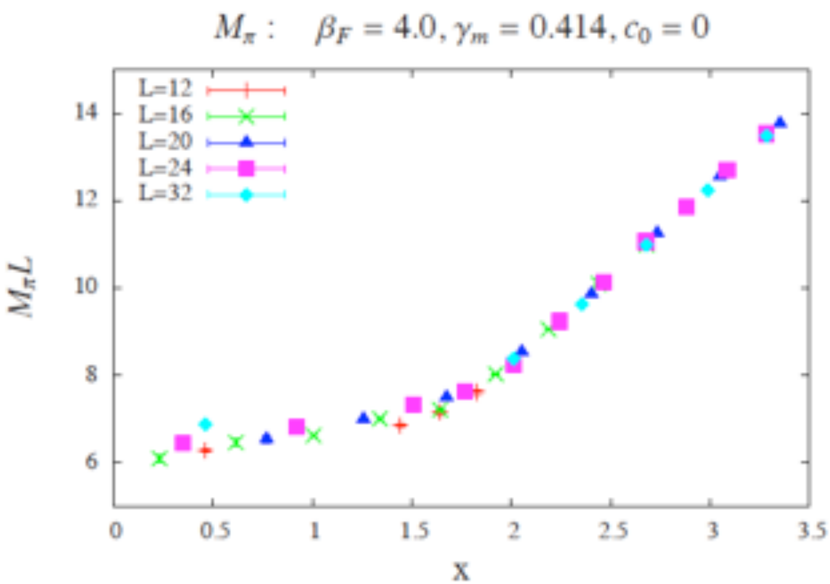


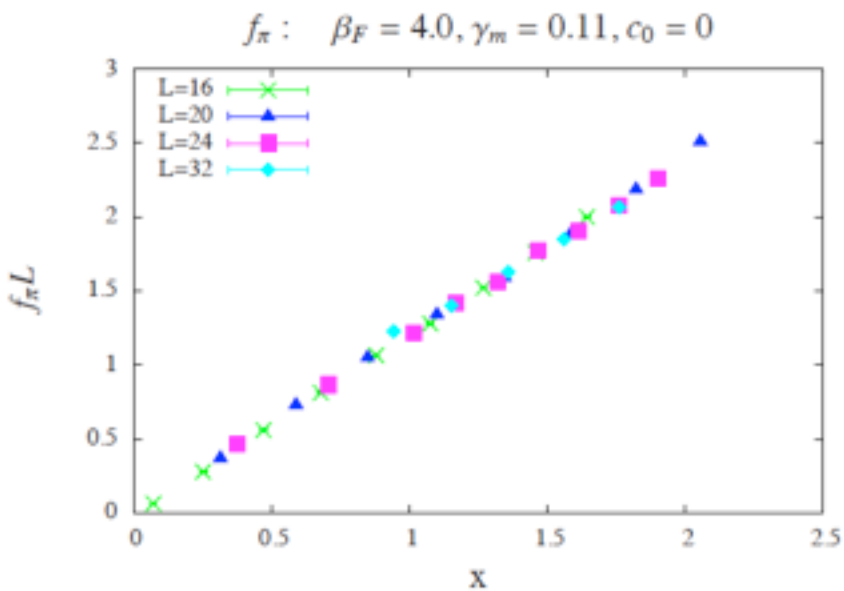


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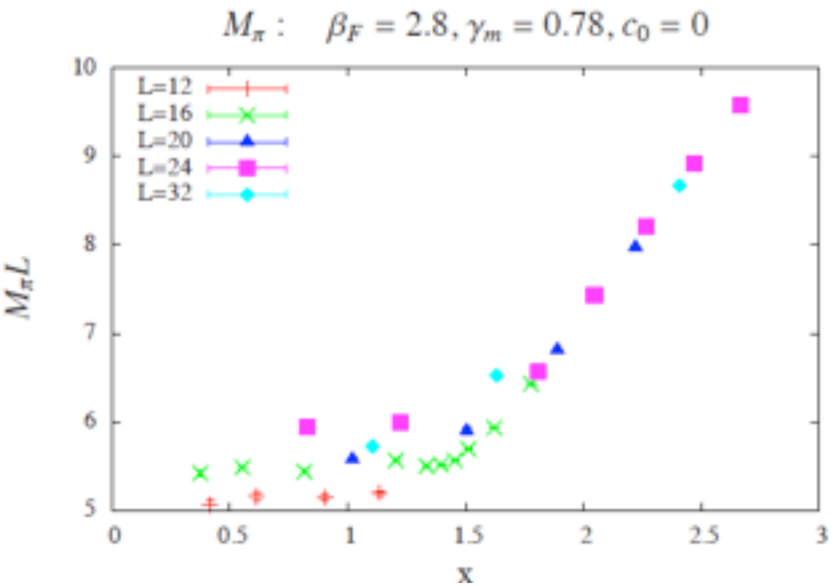
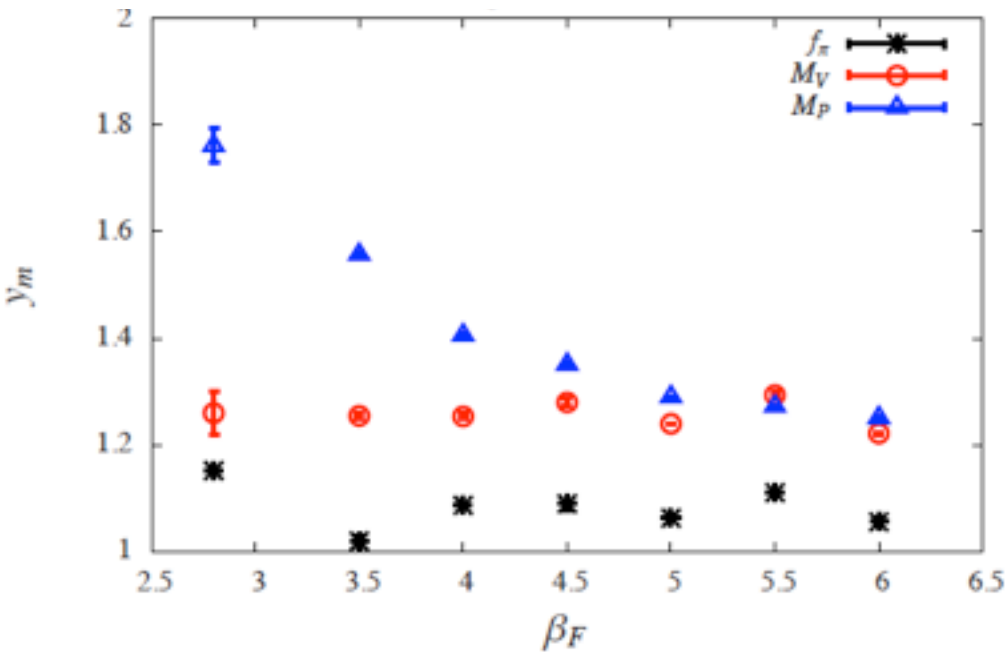



Gets worse at strong coupling!
(β=2.8)

$M_\pi$ : $y_m$=1.78(4) (β=2.8)

# Scaling exponents

"Curve collapse"  for pseudoscalar, vector and $f_\pi$:



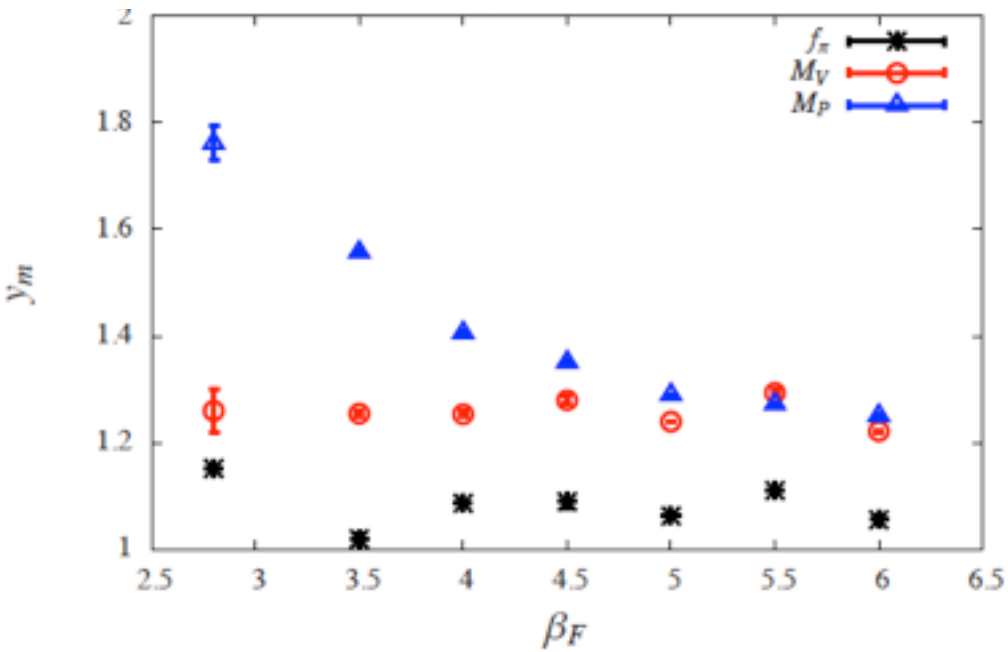

β=2.8 — 6.0
Volumes: $12^3$, $16^3$, $20^3$, $24^3$, $32^3$
$N_T = 2\ N_S$
masses: 0.005 — 0.12
such that x= 0.2 - 5

25 - 35 data points at each β

$M_\pi$, and $M_V$ settle at a common value at β≈6.0
($f_\pi$ is still off)

# Scaling exponents

"Curve collapse"  for pseudoscalar, vector and $f_{\pi}$:

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Possible explanations:

1) $N_f$=12 is not conformal
2) $N_f$=12 is conformal but finite size scaling is strongly affected by an irrelevant operator

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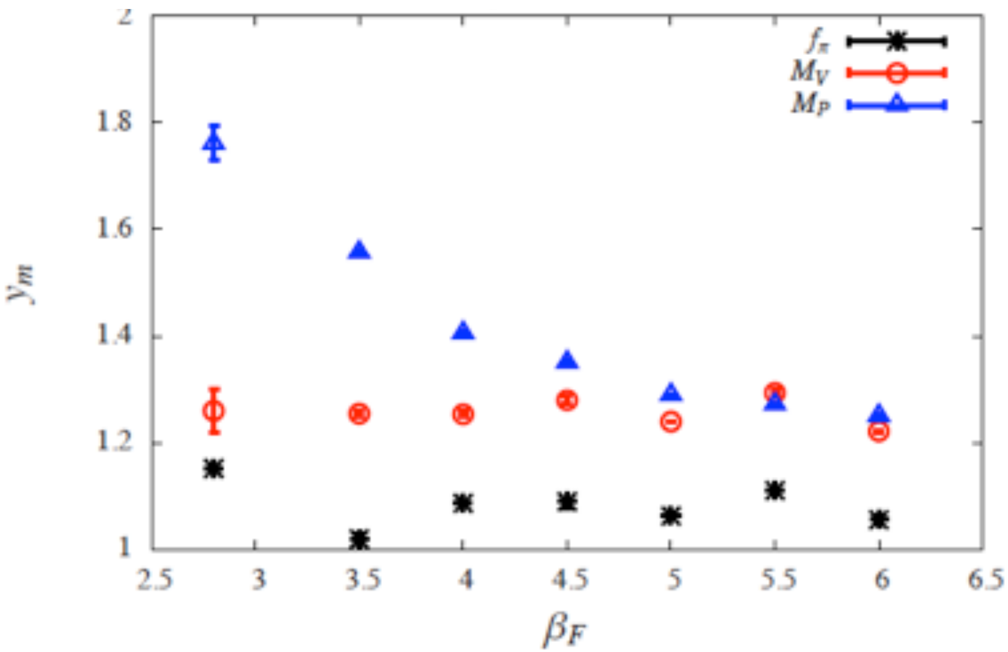


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Possible explanations:

1) $N_f$=12 is not conformal
2) $N_f$=12 is conformal but finite size scaling is strongly affected by an irrelevant operator

# Finite size scaling with a **near-marginal operator**

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

$g_i$ with scaling dimensions $y_i < 0$

$g_0$ (near) marginal, $y_0 \lesssim 0$

Renormalization group arguments in volume $L^3$ predict

$$M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$g_0 \to g_0 m^{\omega}, \quad \omega = -y_0 / y_m \gtrsim 0$$

$$M_H L = f(x, g_0 m^{\omega}), \quad x = Lm^{1/y_m}$$

The scaling function depends on two variables now!

# Corrections to finite size scaling

Physical masses scale as

$$M_H = L^{-1} f(x, g_0 m^{\omega}), \quad \omega = -y_0 / y_m$$

$f(x, g_0 m^{\omega})$ is analytic both in x and $g_0$.

If the $g_0 m^{\omega}$ corrections are small, expand

$$LM_H = F(x)\left(1 + g_0 m^{\omega} G(x)\right)$$

- F(0), G(0) are finite constants
- as $L \to \infty: \quad M_H \propto m^{1/y_m} \to F(x) \propto x, \; G(x) = \text{const}$

Approximate G(x) = c (should be checked) $\to \dfrac{LM_H}{1 + c\, g_0 m^{\omega}} = F(x)$

Need minimization in $y_m$, $\omega$, and $cg_0$

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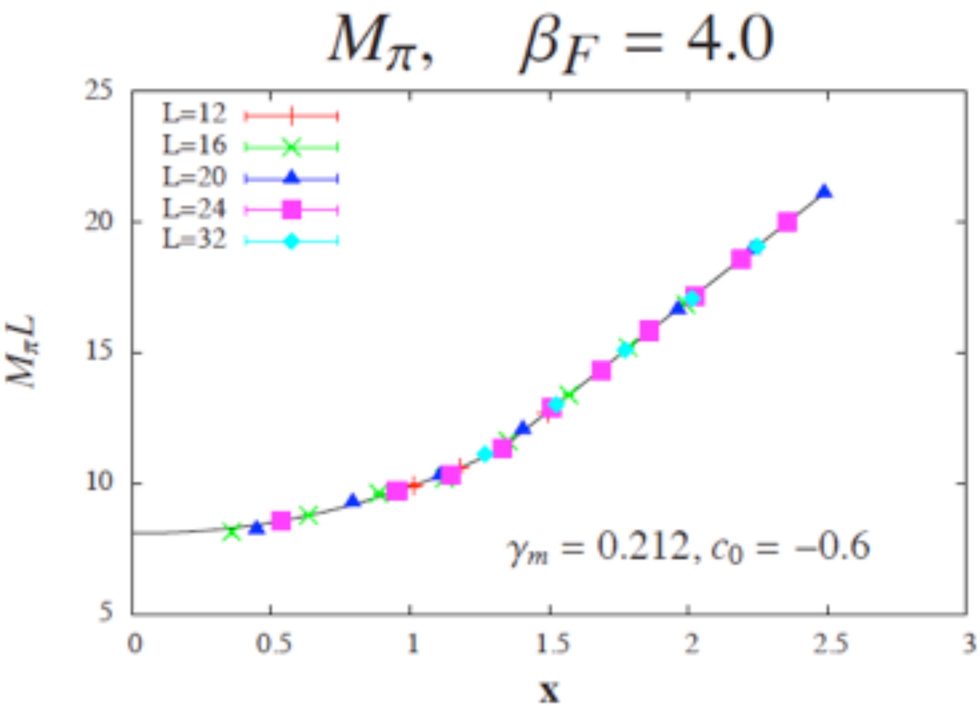
# Scaling test **with** corrections

Curve collapse: 2 parameter, $y_m$ and $c_0$, $y_0$=-0.3 fixed




Fit:
quadratic polynomial at $x < x_0$ ,
linear at $x > x_0$ ,
separation point $x_0$ free
(here $x_0$ = 1.36 )

- Consistent curve collapse both at small and large $x = Lm^{1/y_m}$
  $y_m$=1.212 , $c_0$ = -0.6 ; $\chi^2/\mathrm{dof} = 4.5$
- Cut small x<1.2 points : $y_m$=1.234 , $c_0$ = -0.6; $\chi^2/\mathrm{dof}$ =2.9
- Cut large x>1.3 points : $y_m$=1.184 , $c_0$ = -0.7; $\chi^2/\mathrm{dof}$ = 0.7

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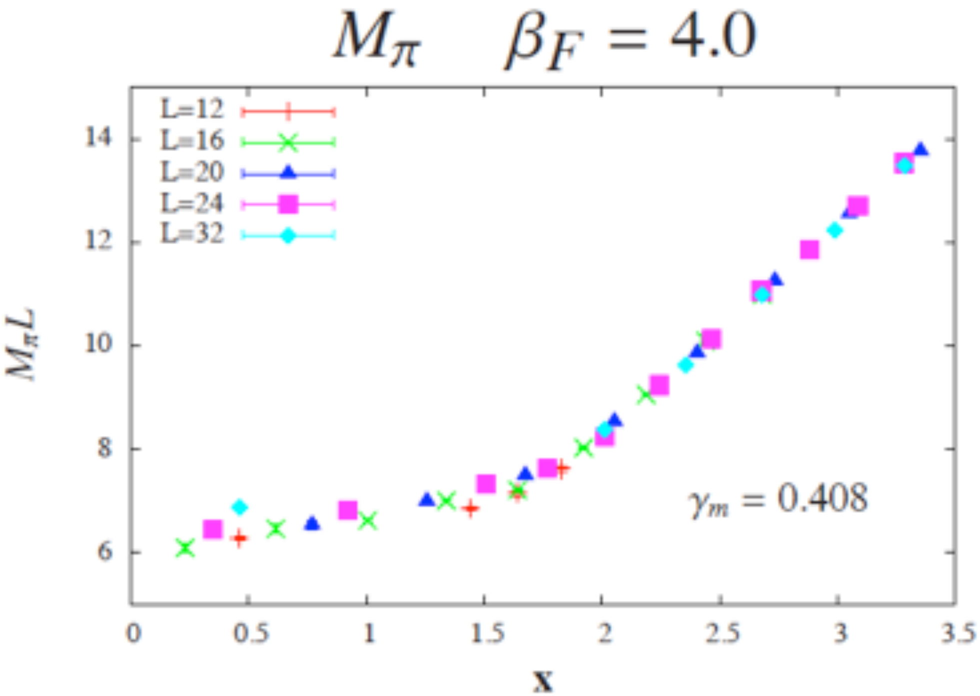


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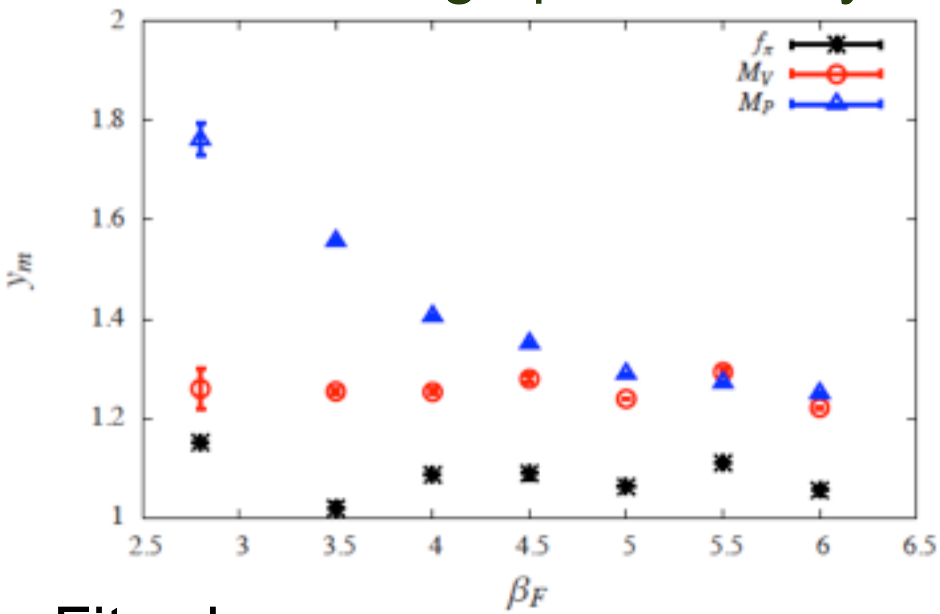
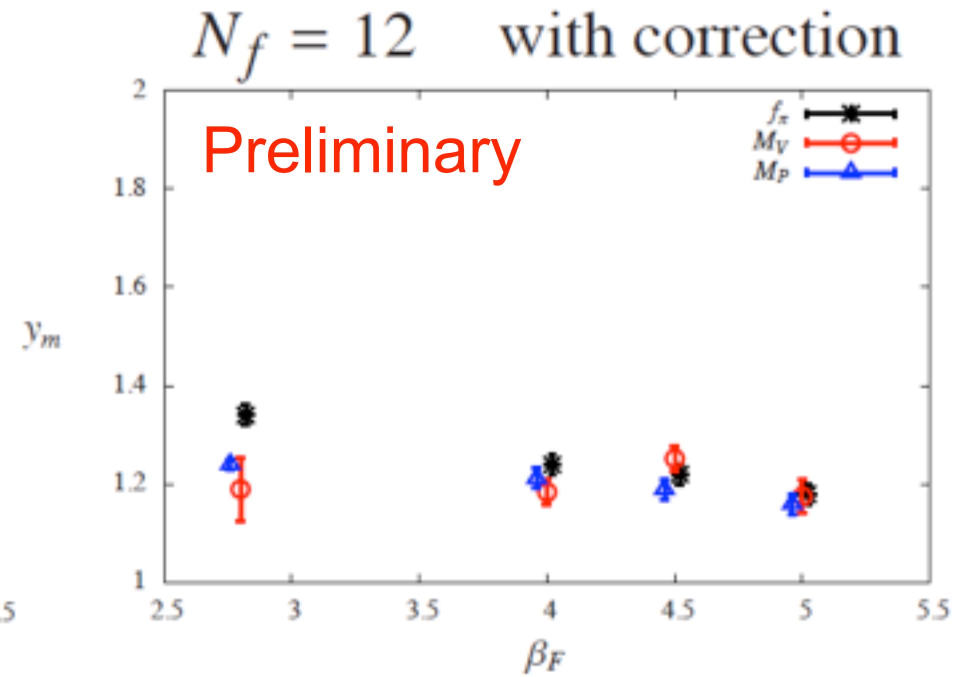
# Scaling exponent **with** corrections

Include all data $M_\pi L$ , $M_V L$ , $f_\pi L$ points

Leading operator only




Fits show

- good curve collapse
- consistent scaling exponent $\gamma_m$=0.20(2)
- but need more data to constrain the 2 parameter fits

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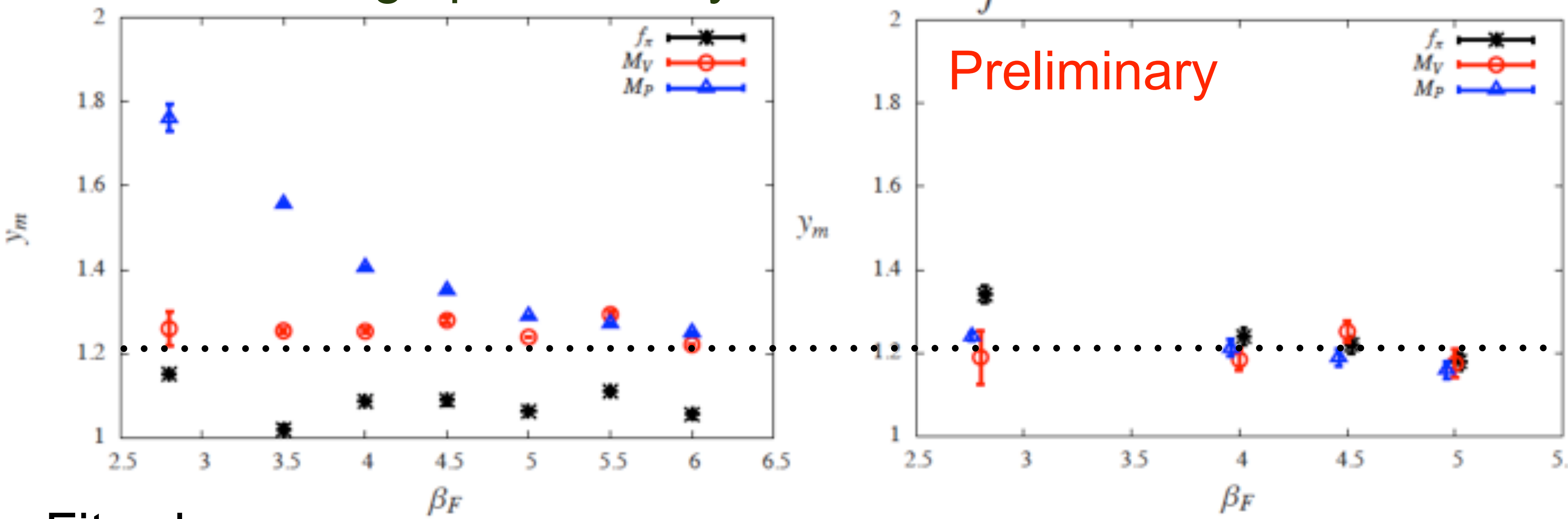
$N_f = 12$ with correction

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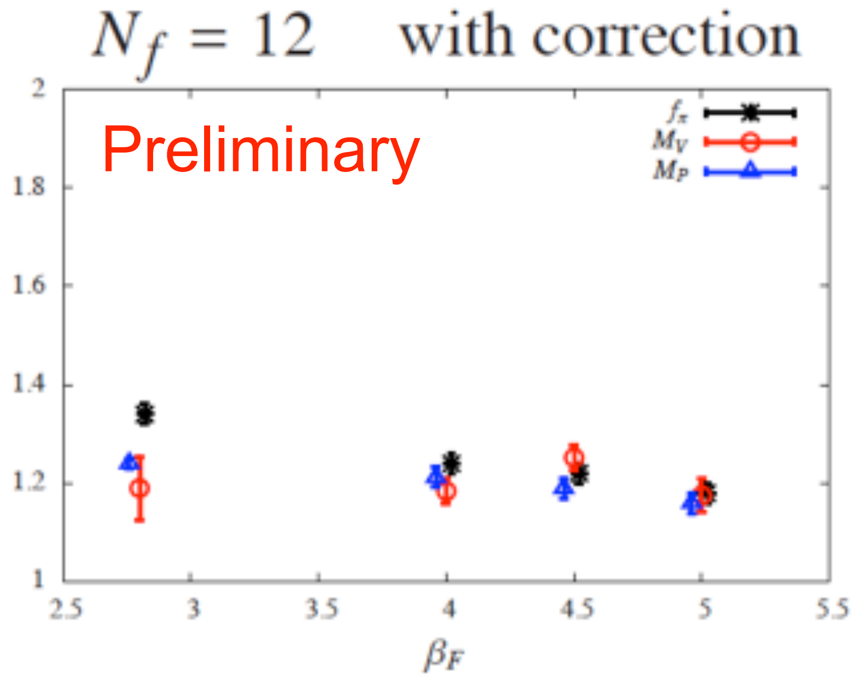



How to make this stronger:

-Combined fit to all beta (same scaling function F(x)!)
-Combined fit to all operators (same exponents)

Preliminary results very promising!

# Summary

Strongly coupled gauge-fermion systems are exciting

- non-perturbative dynamics with unusual properties
- can offer BSM with composite Higgs

Near the conformal window they (could)

- walk : slowly changing gauge coupling
- large anomalous dimension
- dilaton: light scalar! (in progress)

Lattice studies are only starting to understand these systems

# Backup slides

# The exponent $y_0$

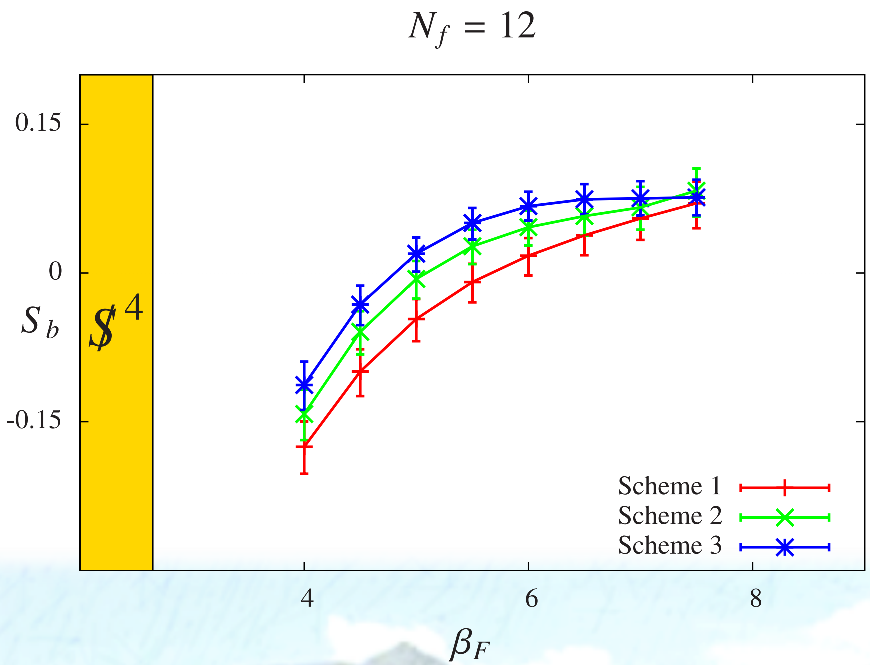
Is $y_0$ ever small?

Perturbatively:

- $N_f$=16 : $y_0$ = - 0.002 (2 loop)
- $N_f$=12 : $y_0$ = - 0.36 — - 0.28 (2 loop /4-loop MS)

Schroedinger funct. studies suggest small $y_0$ in several models

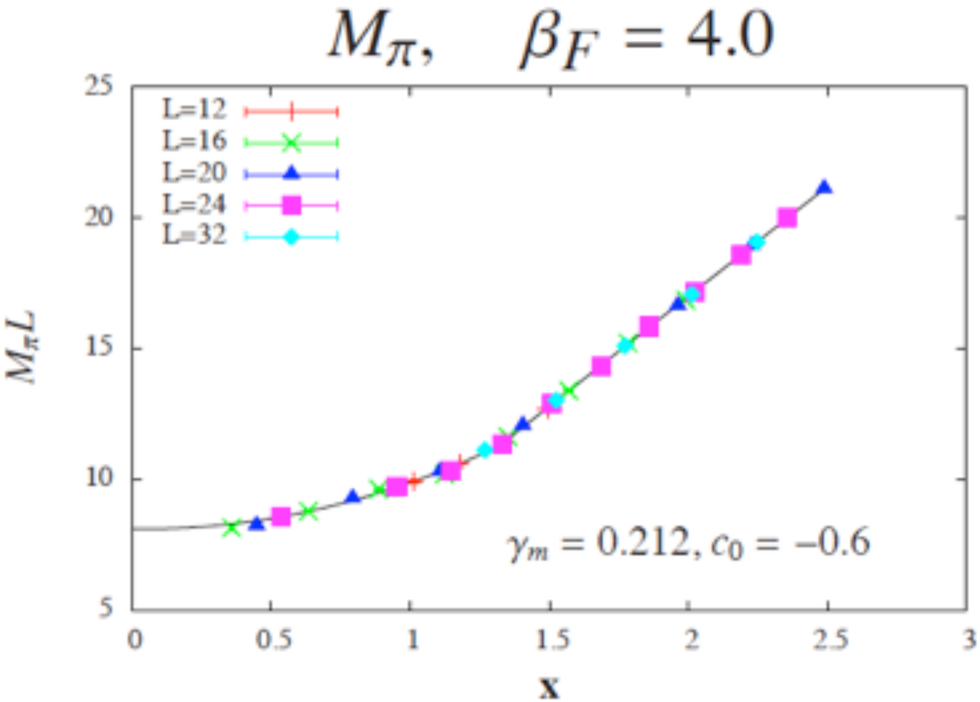
MCRG for $N_f$=12 predicts $y_0 \approx$ - 0.12(4)




Slope of the bare step scaling function predicts $y_0$
G. Petropoulos talk, 15:40 today

# Scaling test **with** corrections

Curve collapse: 2 parameter, $y_m$ and $c_0$, $y_0$=-0.3 fixed

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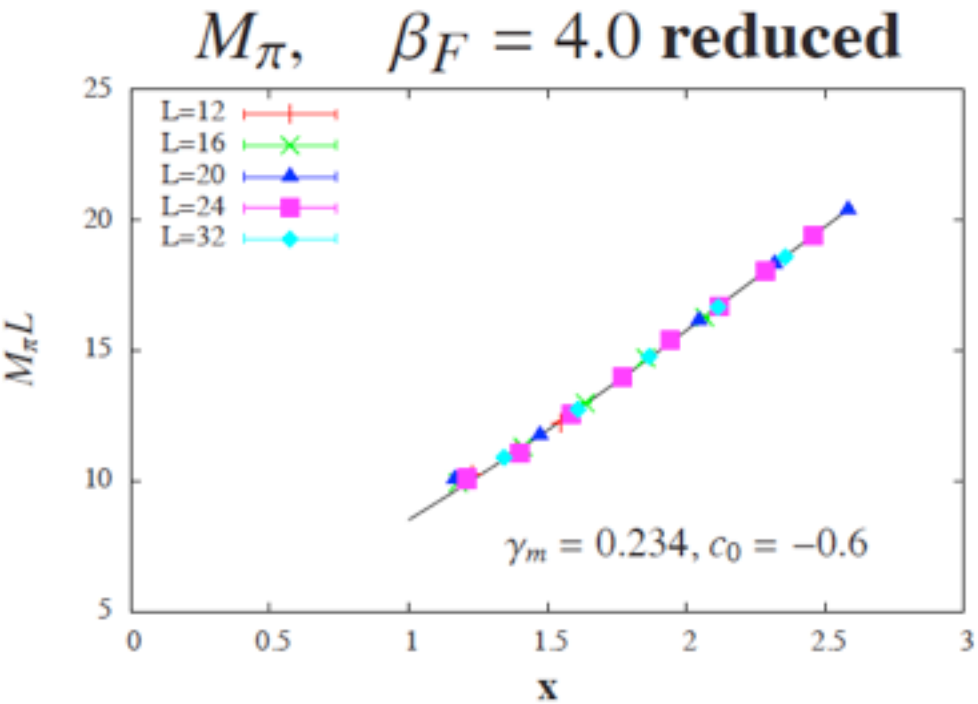


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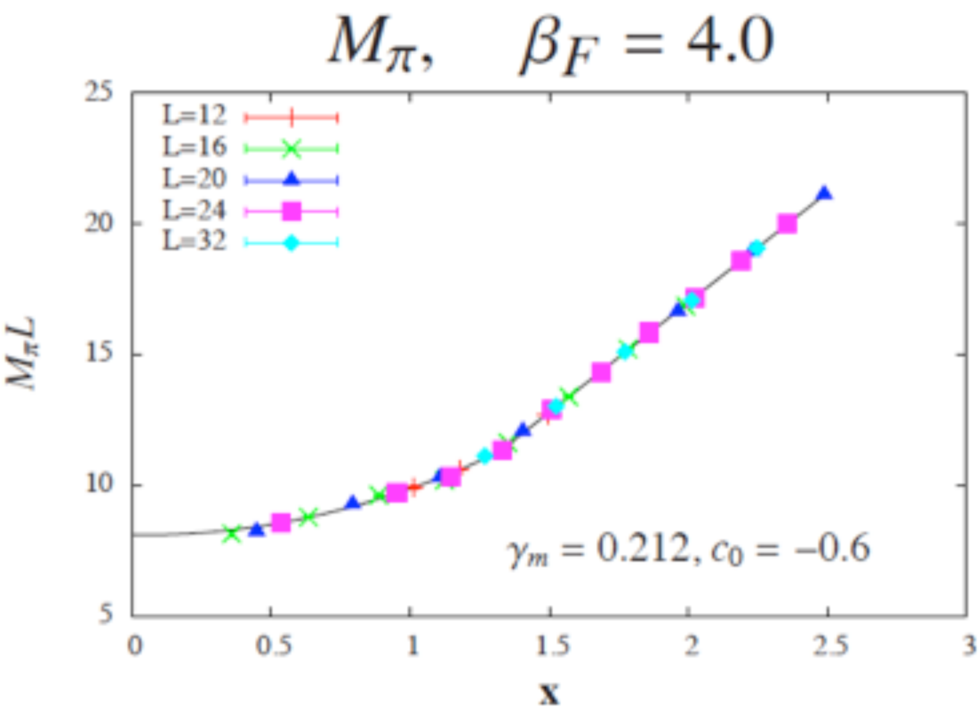
(2 parameter curve collapse, $y_0$=-0.3 fixed)




| | $M_\pi$ | $M_\rho$ | $f_\pi$ |
|---|---|---|---|
| $y_m$ | 1.212(20) | 1.184(25) | 1.24(2) |
| $c_0$ | -0.6 | -0.3 | 1.6 |
| $\chi^2$/dof | 4.5 | 5.1 | 8.5 |
| $y_m$ $c_0=0$ | 1.406(4) | 1.254(5) | 1.084(5) |

- Consistent curve collapse both at small and large $x = Lm^{1/y_m}$
- $y_m$=1.21 consistent for all three observables

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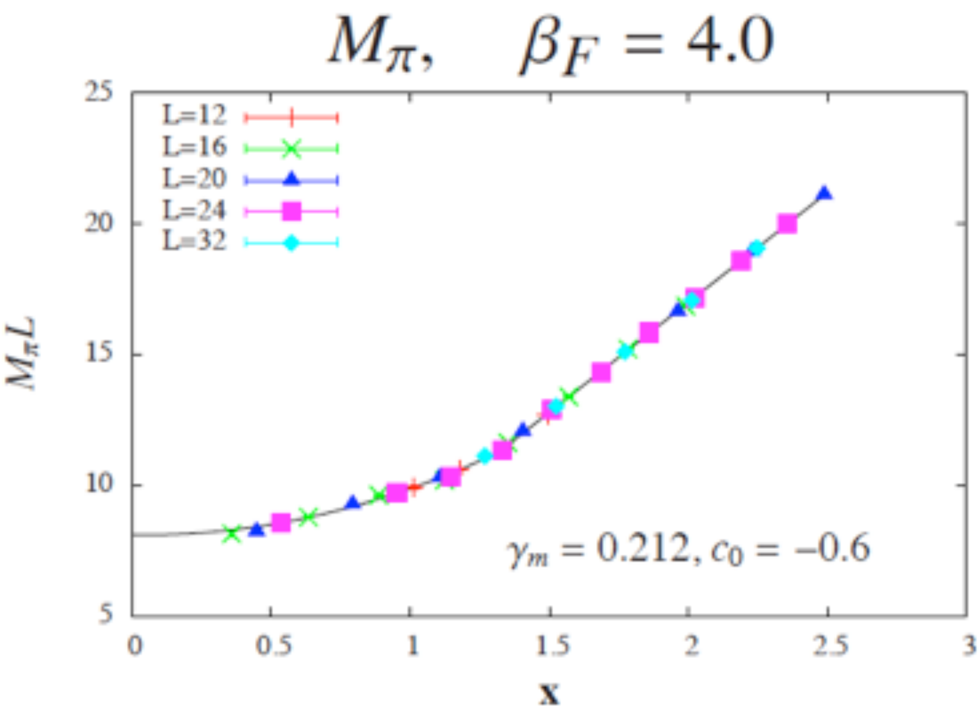


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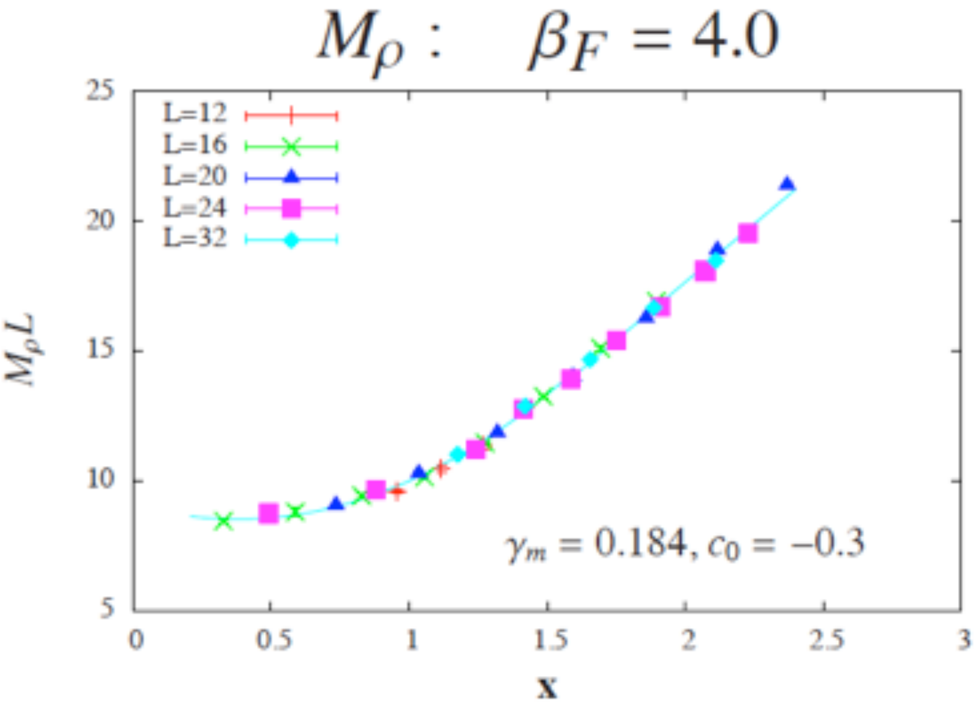


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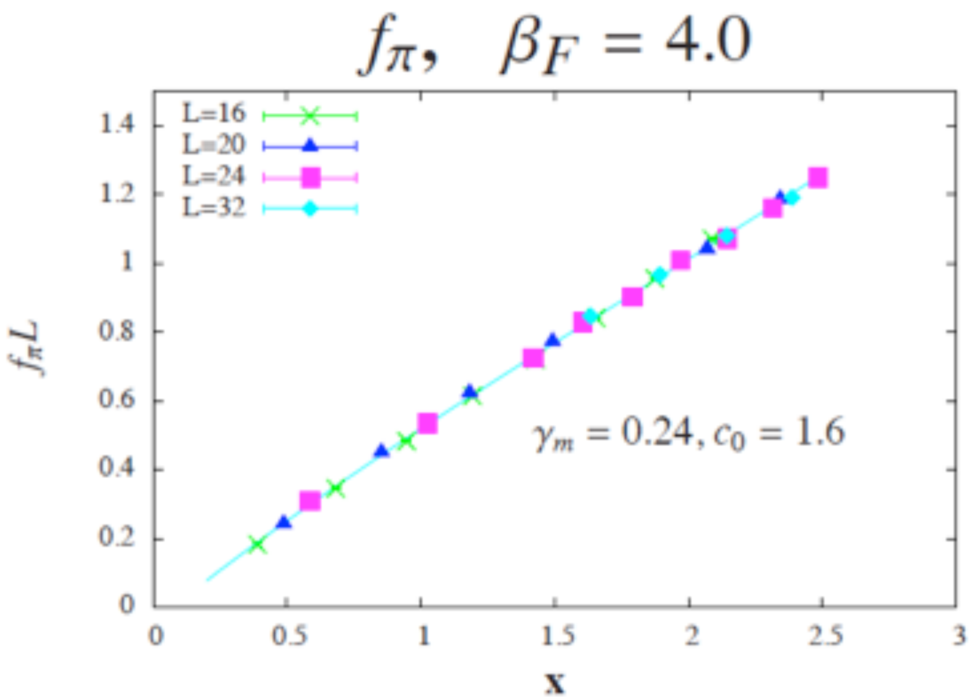


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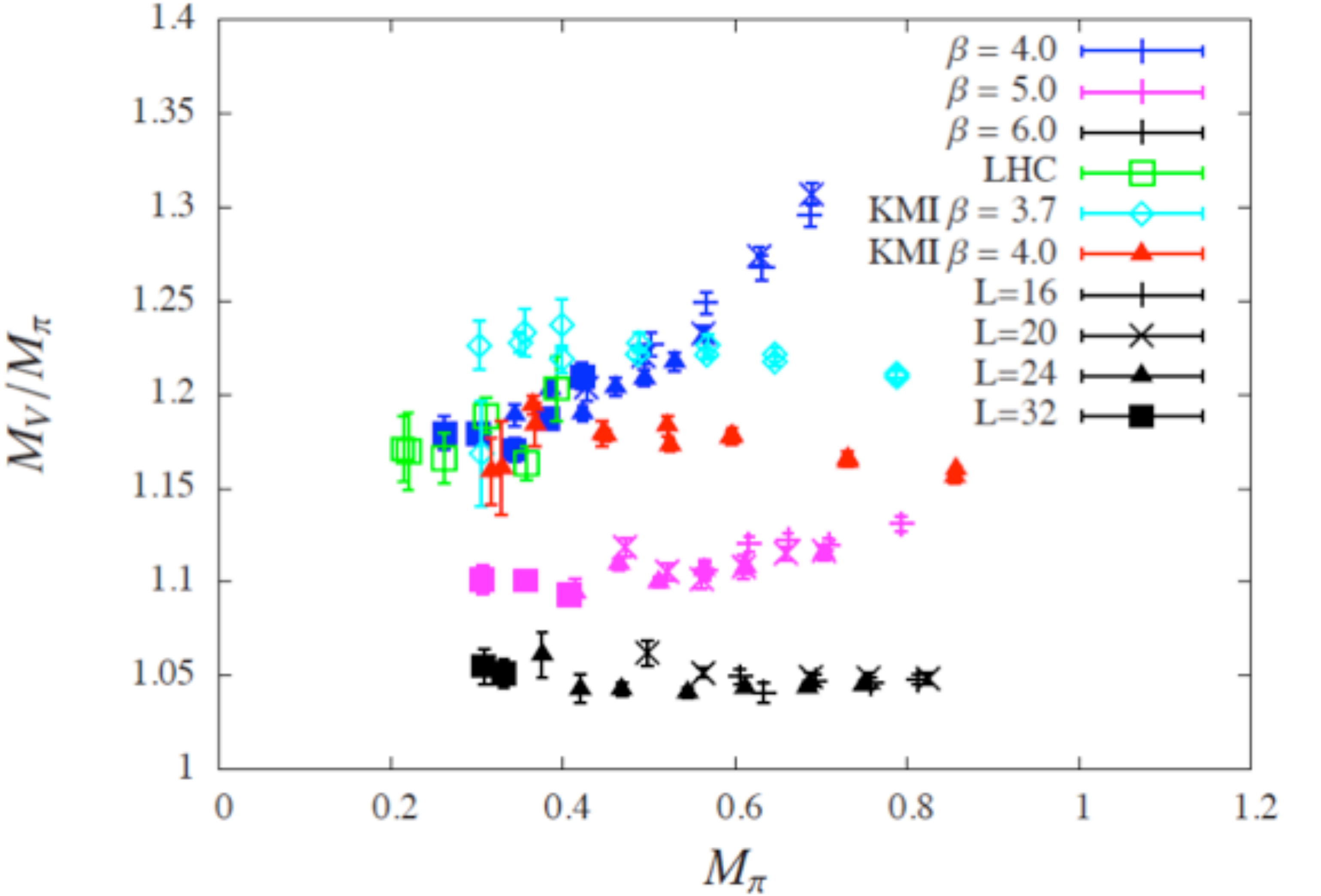
# Meson ratios with LHC and KMI data



# Numerical test

$N_f$=12 flavors nHYP smeared staggered fermions

- gauge coupling: cover a wide range

  $\beta$= 2.8, 4.0, 5.0, 6.0, (3.5, 4.5, 5.5 in progress)

  (Note: $\beta$= 2.8 is near S4b - strongest poss.
  $\beta$= 4.0 is very close to LHC $\beta$=2.2
  $\beta$= 5.5 is the IRFP based on MCRG
  and eigenmodes)

- volumes : $12^3$x24, $16^3$x32, $20^3$x40, $24^3$x48, $32^3$x64

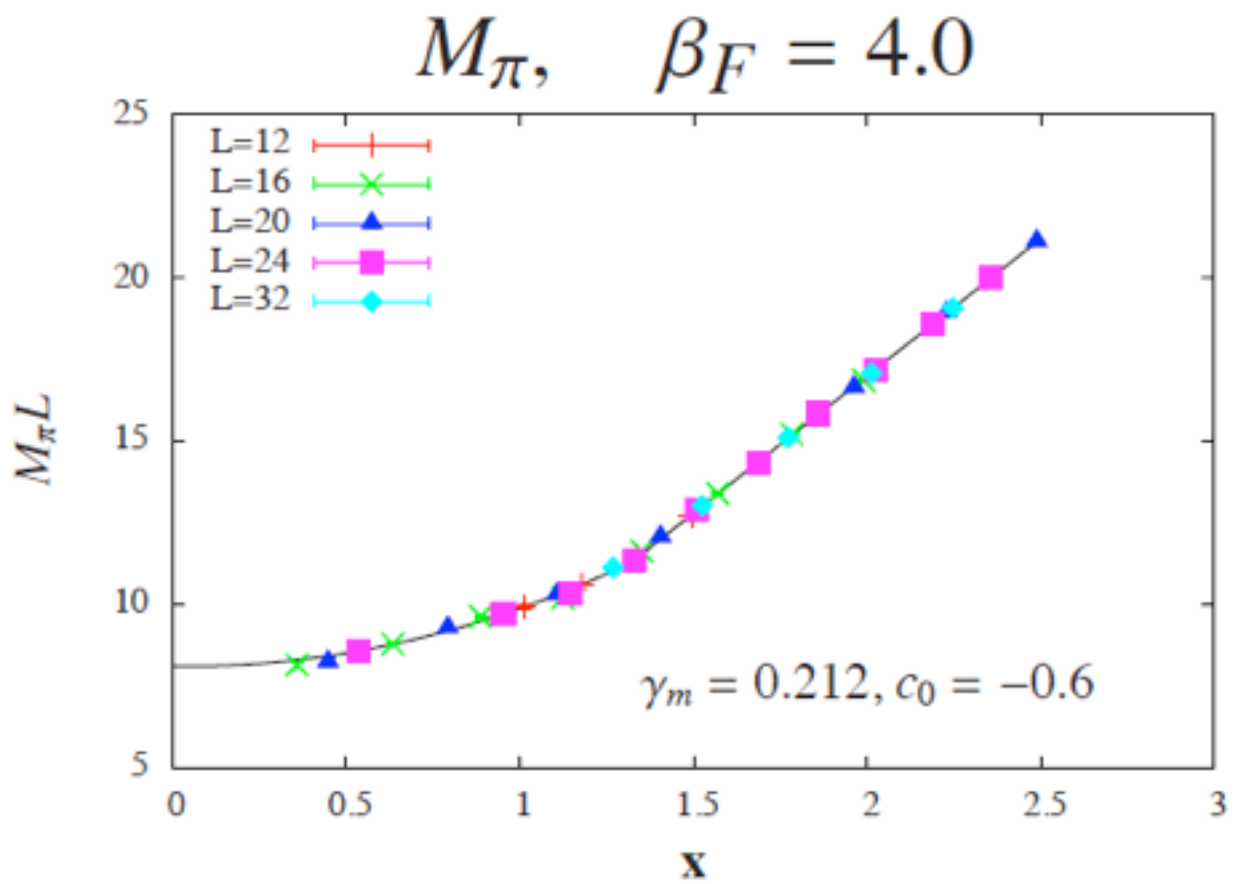
- fermion mass : m=0.01 -- 0.15 ( $x = m^{1/y} L$ = 1- 6 )

25-35 data points at each $\beta$

- operators: pseudoscalar, vector, $f_\pi$

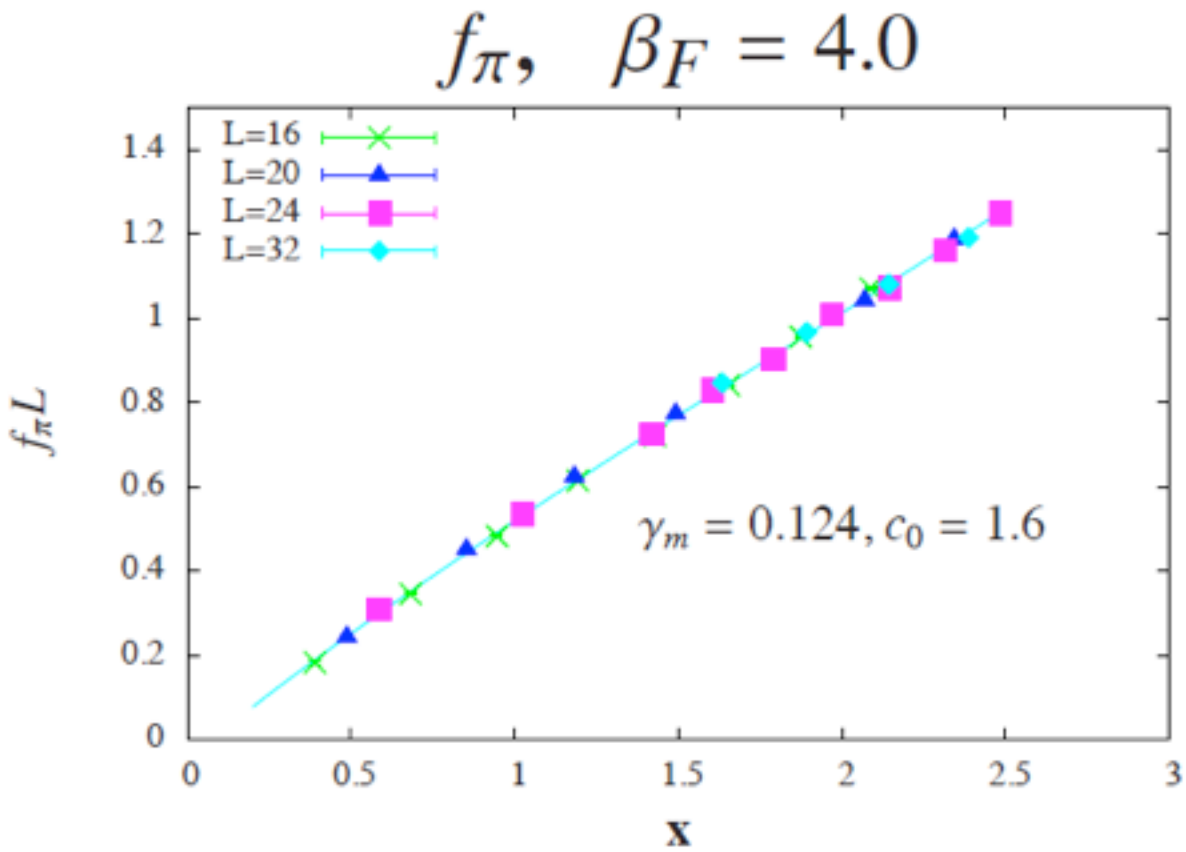
# Fitting forms

$M_\pi$ and $M_\rho$ : fit quadratic at small x, linear at large.



# Fitting forms

$f_\pi$ : 4th order polynomial fit



# Comparing different actions

LHC : 2 stout smeared fermions, Symanzik gauge

KMI : HISQ fermions without Naik, Symanzik gauge

Boulder: nHYP fermions, fundamental+adjoint plaquette gauge

Table: $\gamma_m$ from fits with leading exponent only

| | $6/g^2$ | $\gamma_m$ $(M_\pi)$ | $\gamma_m$ $(M_\rho)$ | $\gamma_m$ $(f_\pi)$ |
|---|---|---|---|---|
| Boulder | 1.4 | 0.76 | 0.26 | 0.15 |
| Boulder | 2.0 | 0.41 | 0.25 | 0.11 |
| LHC | 2.2 | 0.39 | 0.30 | 0.21 |
| Boulder | 2.5 | 0.29 | 0.24 | 0.06 |
| KMI | 3.7 | 0.43 | 0.46 | 0.52 |
| KMI | 4.0 | 0.41 | 0.46 | 0.58 |

Lattice artifacts are not universal!

# Comparing different actions

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Boulder: nHYP fermions, fundamental+adjoint plaquette gauge

Table: $\gamma_m$ from fits with leading exponent only

| | $6/g^2$ | $\gamma_m$ $(M_\pi)$ | $\gamma_m$ $(M_\rho)$ | $\gamma_m$ $(f_\pi)$ |
|---|---|---|---|---|
| Boulder | 1.4 | 0.76 | 0.26 | 0.15 |
| Boulder | 2.0 | 0.41 | 0.25 | 0.11 |
| LHC | 2.2 | 0.39 | 0.30 | 0.21 |
| Boulder | 2.5 | 0.29 | 0.24 | 0.06 |
| KMI | 3.7 | 0.43 | 0.46 | 0.52 |
| KMI | 4.0 | 0.41 | 0.46 | 0.58 |

Lattice artifacts are not universal!